ISSN 2250-0987
Nguyen Van Hieu et al, UNIASCIT, Vol 4 (3), 2014, 450-456

A PESSIMISTIC APPROACH FOR SOLVING A MULTI-CRITERIA DECISION PROBLEM ACCORDING TO PARTIAL INFORMATION ON CRITERIA AND BY DECISION ALTERNATIVE COMPARISON JUDGMENTS

Nguyen Van Hieu[1], Trieu Thi Ni Ni[2]
Faculty of Information Technology,
University of Science and Technology, Da Nang, Viet Nam

***Abstract*—A pessimistic strategy of multi-criteria decision made by using two types of initial information is considered in the paper. The first type is the simplest expert estimates about criteria preferences, including comparison with estimates, interval estimates, etc. The second type is the indirect estimates. According to these estimates, the experts do not evaluate the criteria, but they compare some decision alternatives. Simple algorithms for solving such the decision problems are developed. They reduce to solving a number of linear programming problems. Main results are explained and illustrated by numerical examples.**

***Keywords*—multi-criteria decision problem, linear programming, convex set, weights, pessimistic strategy.**

I. INTRODUCTION

Multi-criteria decision making (MCDM) methods are an important set of tools for searching the "best" decision alternative (DA) among its set of DA`s or ranking the DA`s with respect to multiple, usually conflicting attributes, under uncertainty and complexity. There are many methods for dealing with multiple criteria. In accordance with a large part of the MCDM methods the different objectives or criteria are aggregated to one overall objective function which is called a global criterion sometimes. Then the "best" DA can be found by using one objective function. The result in this case strongly depends on how the objectives were aggregated.

The most widely used method of aggregating is the linear weighted-sum approach. The weight can reflect the relative importance of a criterion to the decision. The weights are usually determined on subjective basis. They represent the opinion of a single decision maker (DM) or synthesize the opinions of a group of experts using a group decision technique. By choosing the certain weights for the different objectives, the preferences of the decision maker (DM) can be taken into account.

Although the linear weighted-sum approach is rather simple, it has a number of shortcomings. First of all, it is impossible to locate solutions at non-convex parts of the Pareto-front by using this approach. As the finite set of DA`s is always non-convex, then the approach can give incorrect results in this case.

Some shortcomings of the linear weighted-sum approach can be overcome by taking the nonlinear maximin approach. The maximin method is based upon a strategy that tries to avoid the worst possible performance, maximizing the minimal performing criterion [9]. It can be used only when all criteria are compared so that they can be measured on a common scale, which is a limitation [1]. Nevertheless, this is an interesting approach [11] applying pessimistic decision making. Therefore, it will be researched in the presented paper.

One of the most important questions in frameworks of MCDM methods is how to model the preferences of a decision maker (DM) and how to represent the elicited weights of criteria. Some MCDM methods assume that the weights are precise due to completeness of preference information elicited from the DM. However, the preference information is usually imprecise and incomplete in the real situations. One of the approaches for representing different types of imprecision is to consider a set of weights produced by possible judgments provided by experts or by the DM [1, 10, 11]. Park and Kim [3], Kim and Ahn [4] formalized different kinds of judgments or statements by means of a number of the linear inequalities for weights of criteria. Due to linearity, these inequalities form a convex polytope of vectors of weights and this fact gives the possibility to use linear programming for computing a measure for ranking the DA`s. It should be noted that such the model of preference information representation is rather general and incorporates different frameworks of representation's uncertainty, including Dempster-Shafer theory [5], etc.

Methods for processing the sets of the elicited weights of criteria are quite different. Some methods assume the uniform distribution of vectors of weights inside the set. Usually these methods use one precise vector of weights as an equivalent representation of the set. However, the assumption of uniformly distributed vectors of weights is rather strong and there are a number of the arguments against this assumption. Another part of the methods works with the whole set of weights by applying the optimization problems for computing some bounds for the overall objective function or for reducing the set of non-dominated or Pareto-optimal DA`s. These methods do not use any assumption about a probability distribution of weights and it is assumed that every distribution is possible. One of such the methods will be used in the presented paper for solving the decision problem.

So, a MCDM method using the maximin strategy is proposed and studied in the paper. Moreover, the DM preferences are formalized by means of a convex set of weights or of a polytope produced by linear inequalities. In this case, the main feature or difficulty of the proposed method is that ranking the DA`s is based on solving optimization problems with the non-linear objective function and linear constraint. In the second part of the paper, a new type of DM judgments is introduced. The DM does not estimate or compare criteria, but he(she) compares some DA`s.

This leads to non-linear inequalities in constraints and more complicated optimization problems. Nevertheless, it is shown in the paper that non-linear optimization problems with non-linear objective functions and non-linear constraints can equivalently be replaced by a finite set of linear programming problems. Simple algorithms are proposed for realizing the MCDM method.

The paper is organized as follows. In section 2, the mathematical statement of problem is given and an aggregated of criteria is proposed. A method for replacing the non-linear optimization problems with linear constraints by a set of linear programming problems is given in section 3. Comparative judgment and their processing in MCDM are investigated in section 4.

II. INITIAL INFORMATION AND CRITERION

Suppose that there is a set of DA`s $A=\{A_1,...,A_n\}$ consisting of n elements. Moreover, there is a set of criteria $C=\{C_1,...,C_r\}$ consisting of r elements. Let $w=\{w_1,...,w_r\}$ be a vector of "weight" or importance measures of r criteria C satisfying the following conditions:

$$\sum_{i=1}^{r} w_i = 1,\ \forall i: i=1,...,r,\ w_i \geq 0 \qquad (1)$$

In broad terms, Park and Kim [3], Kim and Ahn [4] distinguish between the following approaches to the elicitation of attribute weights: weak ranking $w_i \geq w_j$; strict ranking $w_i - w_j \geq \lambda_i$; ranking with multiples $w_i \geq \lambda_i w_j$; interval form $\lambda_i \leq w_i \leq w_j + \varepsilon_i$; ranking of differences $w_i - w_j \geq w_k - w_l$. Here $\lambda_i \geq 0,\ \varepsilon_i \geq 0$.

The above mentioned statements correspond to linear constraints on attribute weights. This implies that every statement or judgment (say the k-th judgment) provided by a DM produces a r-dimensional convex polytope denoted by W_k.

Denote W^0 the set of weights produced by conditions (1). This set is the unit simplex. Suppose that a set of N possible judgments provided by the DM supplemented by conditions (1) produce a convex polytope denoted by W such that $W = W_1 \cap W_2 \cap ... \cap W_N \cap W^0$. In other words, every point $w \in W$ of the polytope simultaneously satisfies every judgment and (1).

A method for aggregating and processing the above incomplete information totally depends on the criterion of decision making. In general, a large part of decision methods consists of aggregating the different local criteria from the set C into a function called a global criteria, which has to be maximized. According to these methods, global criteria by a finite set A of DA's can be represented as follows:

$$\mathrm{F}(\mathbf{w},\mathbf{u}_k) \rightarrow \max_{\mathrm{A}} \qquad (2)$$

Here $\mathbf{w}=(w_1,...,w_r)$ is the vector of "weights" or importance measures of criteria.

$\mathbf{u}_k=(u_{1k},...,u_{rk})$, $k=1,...,n$, is the vector of "weights" or utilities of the k-th DA with respect to every criterion from $\{C_1,...,C_r\}$. F is a function allowing us to combine the "weights" of criteria and DA's in order to get a final measure of "optimality" of every DA. In particular, one of the most widely applied criteria is the linear function F, i.e.,

$$\mathrm{F}(\mathbf{w},\mathbf{u}_k) = \sum_{i=1}^{r}\left(w_i \cdot u_{ik}\right) \qquad (3)$$

However, in spite of the wide application of the linear function for combining criteria in multi-criteria decision making, it has been shown in the literature that this function has a number of shortcomings. Therefore, in case of pessimistic decision making, the maximum technique can be applied, for which the following holds

$$\mathrm{F}(\mathbf{w},\mathbf{u}_k)=\min_{i=1,\dots,r}\left(w_i\cdot u_{ik}\right) \qquad (4)$$

It is obvious that, by having only partial information about w, we can only get own information about F. Suppose that the vector w takes values from the sets W. Due to convexity of W, we can say that the function F belongs to some interval having the lower bound L_k and upper bound U_k such that

$$\mathrm{L}_k=\inf_{w\in W}F\left(w_i\cdot u_k\right),\qquad \mathrm{U}_k=\sup_{w\in W}F\left(w_i\cdot u_k\right)$$

The choice of the "best" DA can be based on comparison of intervals [L_k; U_k] for all k =1,...,n. There exist a lot of methods for comparison. We propose to use the most justified method based on the so-called caution parameter [1, 10, 11] or the parameter of pessimism η ä [0,1]. According to this method, the "best" DA among possible ones should be chosen in such a way that makes the convex combination $\eta\cdot \mathrm{L}_k+(1-\eta)\cdot \mathrm{U}_k$ achieve its maximum. If $\eta=1$, then we analyze only lower bounds for F(**w**, $\mathbf{u}_k$) and make pessimistic decision. If $\eta=0$, then we only analyze upper bounds for F(**w**, $\mathbf{u}_k$) and make optimistic decision.

The lower and upper bounds can be found by solving the linear programming problems when the global criterion is linear. However, the problem becomes complicated when we use a non-linear criterion, for instance, the maximin technique given by (4). In this case, we have to solve the following optimization problems with linear constraints and non-linear objective functions:

$$\mathrm{L}_k=\inf_{\mathbf{w}\in \mathrm{W}}\min_{j=1,\dots,r}\left(w_j\cdot u_{jk}\right),\ \ \mathrm{U}_k=\sup_{\mathbf{w}\in \mathrm{W}}\min_{j=1,\dots,r}\left(w_j\cdot u_{jk}\right).$$

III. METHODS FOR SOLVING THE PROBLEMS

A. *The First Problem*

Our aim is to solve the following optimization problem:

$$L_k=\inf_{w\in W}\min_{j=1,\dots,r}(w_j\cdot u_{jk}). \qquad (5)$$

Proposition 1: Suppose that the set W_G is formed by the constraints $w\in W$ and the following constraints with the variable G:

$$G=w_i\cdot u_{ik},$$
$$G\le w_j\cdot u_{jk},\ j=1,\dots,r,\ j\ne i.$$

If the set W_G has M extreme points $(w_1^{(i)},w_2^{(i)},\dots,w_r^{(i)},G^{(i)})$, i = 1..M, then the optimal value L_k in problem (5) can be found as follows:

$$L_k=\min_{i=1,\dots,M}\min_{j=1,\dots,r}\{w_j^{(i)}\cdot u_{jk}\}.$$

Proof. Let us introduce a variable $G=\min_{j=1,\dots,r}\left(\mathrm{w}_j\cdot u_{jk}\right)$. Then the above problem can be rewritten as follows:

$$L_k=\inf_{\mathrm{w}}\mathbf{G}$$

Subject to $w\in W$ and

$$G\le \mathrm{w}_j\cdot u_{jk},\ \ j=1,\dots,r.$$

We have obtained the linear programming problem having r + 1 variables. However, it can be seen from the above that the problem does not have any solution because by minimizing the objective function, the variable **G** will unrestrictedly decrease. How to restrict these variables? It follows from the definition of the introduced variable G that the optimal value of G is $\mathrm{w}_j\cdot u_{jk}$. This implies that the optimal solution corresponds to the equality in one of the constraints $G\le \mathrm{w}_j\cdot u_{jk}$. This equality restricts the variable G. Therefore, we have to solve r linear programming problems such that the i-th problem has one constraint $G=\mathrm{w}_i\cdot u_{ik}$ instead of $G\le \mathrm{w}_i\cdot u_{ik}$. Suppose that the i-th problem has the optimal solution $G^{(i)}$. Then

$$L_k=\min_{i=1,\dots,r}\mathrm{G}^{(i)}$$

Note that the i-th problem is linear. This implies that the optimal solution to the above linear programming problem can be found at one of the extreme points extr(W_G) of the convex set W_G produced by the linear constraints. The procedure for computing extreme points is well-known and is reduced to solving a finite set of linear programming problems. Note that the extreme points are determined only by the set W_G and do not depend on the objective function. This implies that the lower belief function can be computed by substituting the extreme points extr(W_G) into objective functions of r linear programming problems. As a result, we have r * M linear programming problems, where M is the total number of extreme points of W_G. Therefore, the initial problem (5) can be rewritten as (6).

Corollary 1. Suppose $u_{jk}\ge 0$ for all $j=1,\dots,r$. If at least one element of at least one extreme point is 0, then there holds $L_k=0$.

Proof. It directly follows from (6).

B. *The Second Problem*

Our aim now is to solve the following optimization problem:

$$U_k = \sup_{w\in W} \min_{i=1,\dots,r} \{w_i \cdot u_{ik}\}. \quad (7)$$

Proposition 2: The optimal value U_k in problem (7) coincides with the optimal value U_k in the following problem:

$$U_k = \sup_w G \quad (8)$$

Subject to $w \in W$ and

$$G \le w_j \cdot u_{jk},\ j = 1,\dots,r$$

Proof. The proof is obvious if we introduce the optimization variable $G = \min_{j=1,\dots,r} (w_j \cdot u_{jk})$.

The problem (7) is linear and has r + 1 variables.

Example 1: Take the problem of selecting the best web site for online advertising as a typical example, a list of criteria and their detailed description can be found in [6]. Simply, we use three criteria. The first criterion is the impression's rate. It makes advertisers a sense of the traffic at the site. The second criterion is the cost referred to full banners placed at the top of a web page. The third criterion is the content. It is measured by assessing the variety and reliability of the information.

Sites are analyzed and evaluated for online advertising. The corresponding rates of the sites with respect to every criterion are shown in Table 1. Here a 5-point scale (1-5) is used for rating the sites.

TABLE 1
RATES OF SITES WITH RESPECT TO THREE CRITERIA

	Impression rate	Cost	Content Quality
Site 1	5	3	2
Site 2	2	1	4
Site 2	3	5	3
Site 4	1	2	5

It is known that the impression's rate is more important than the content quality, but more than or equal twice of the content quality is more considerable than the former. It means that $w_1 \ge w_3$ and $w_1 \le 2w_3$. Moreover, the cost is important, but not very important [6]. This fact can be formalized as $0.5 \le w_1 \le 0.75$.

In order to compute the lower bounds L_k, k =1,…,4, we have to find extreme points of the polytope produced by the constraints:

$$\begin{cases} w_1 \ge w_3, \\ w_1 \le 2 \cdot w_3, \\ w_2 \ge 0.5, \\ w_2 \le 0.75, \\ w_1 + w_2 + w_3 = 1, \\ w_i \ge 0, \forall i = 1,\dots,3. \end{cases}$$

There are four extreme points (the element of an extreme point corresponding to G is not written here because it is not used later).

$$\left(\tfrac{1}{6}, \tfrac{3}{4}, \tfrac{1}{12}\right), \left(\tfrac{1}{8}, \tfrac{3}{4}, \tfrac{1}{8}\right), \left(\tfrac{1}{4}, \tfrac{1}{2}, \tfrac{1}{4}\right), \left(\tfrac{1}{3}, \tfrac{1}{2}, \tfrac{1}{6}\right).$$

These points were found by solving systems of three equation constructed from the above constraints.

There is another extreme point associated with the constraints containing G. This point is obtained by solving the following system of equations.

$$G = w_i \cdot u_{ik}, i = 1, 2, 3,$$
$$w_1 + w_2 + w_3 = 1$$

In particular, if k =1, then the system of equation is

$$\begin{cases} G = w_1 \cdot 5, \\ G = w_2 \cdot 3, \\ G = w_3 \cdot 2, \\ w_1 + w_2 + w_3 = 1. \end{cases}$$

Hence, we get the fifth extreme point $\left(\frac{6}{31}, \frac{10}{31}, \frac{15}{31}\right)$. The similar points can be computed for k =2, 3, 4.

$$\left(\tfrac{2}{7}, \tfrac{4}{7}, \tfrac{1}{7}\right), \left(\tfrac{5}{13}, \tfrac{3}{13}, \tfrac{5}{13}\right), \left(\tfrac{10}{17}, \tfrac{5}{17}, \tfrac{2}{17}\right).$$

By having the extreme points and by using Table 1, we can find L_k. For instance, L_1 is

$$\begin{aligned} L_1 = \min\{&\min(\tfrac{1}{6}\cdot 5, \tfrac{3}{4}\cdot 3, \tfrac{1}{12}\cdot 2), \\ &\min(\tfrac{1}{8}\cdot 5, \tfrac{3}{4}\cdot 3, \tfrac{1}{8}\cdot 2), \min(\tfrac{1}{4}\cdot 5, \tfrac{1}{2}\cdot 3, \tfrac{1}{4}\cdot 2), \\ &\min(\tfrac{6}{31}\cdot 5, \tfrac{1}{2}\cdot 3, \tfrac{1}{6}\cdot 2), \min(\tfrac{1}{3}\cdot 5, \tfrac{1}{2}\cdot 3, \tfrac{1}{6}\cdot 2)\} \\ = \tfrac{1}{6}. \end{aligned}$$

The values F_k, with k =2, 3, 4 can be found in the same way

$$F_2 = \frac{1}{4}, F_3 = \frac{1}{4}, F_4 = \frac{1}{8}.$$

In order to compute the upper bounds U_k, k =1,…,4, we have to solve the linear programming problems (8). For instance, U_1 is computed from the following problem

$$U_1 = \sup_{w} G,$$

subject to (9) and

$$\begin{cases} G \le w_1 \cdot 5, \\ G \le w_2 \cdot 3, \\ G \le w_3 \cdot 2 \end{cases}$$

Hence $U_1 = \frac{1}{2}$. By replacing three last constraints, we obtain upper bounds

$$U_2 = \frac{4}{7}, U_3 = \frac{3}{4}, U_4 = \frac{1}{3}.$$

Finally, we have four intervals

$$\left[\frac{1}{6}, \frac{1}{2}\right], \left[\frac{1}{4}, \frac{4}{7}\right], \left[\frac{1}{4}, \frac{3}{4}\right], \left[\frac{1}{8}, \frac{1}{3}\right].$$

Hence, It can be concluded that's the third site is optimal. By using the obtained intervals, we can also rank the sites as follows:

3 f 2 f 1 f 4.

IV. COMPARATIVE JUDGMENTS ABOUT DA`S

The statements or judgments studied above do not take into account a possible choice by DM between two DA`s. For instance, the DM might suppose that the **i**-th DA is more preferable than the **l**-th DA. How to use this information for reducing the set of possible optimal DA`s? We study a case when only one comparative judgment is available.

If we use the maximin technique (4), then the above comparative judgment can be represented as an additional constraint of the form:

$$\min_{j=1,..,r} (w_j \cdot u_{ji}) \ge \min_{j=1,...,r} (w_j \cdot u_{j,l}) . \qquad (9)$$

However, the main difficultly here is that constraint (9) is non-linear and we have a more complicated case of no-linear objective function (5), (7) and non-linear constraints (9). Let us try to tackle this difficulty and to replace the non-linear optimization by linear ones. Note that (9) can be rewritten as the set of the inequalities

$$w_t u_{t,i} \ge \min_{j=1,...,r} (w_j \cdot u_{j,l}), t = 1,..., r. \qquad (10)$$

In other words, we replace the non-linear left-hand side of inequality (9) by the set of inequalities whose left-hand sides are linear. Every inequality in (10) has to be satisfied. So, we pay for overcoming the non-linearity of the left-hand side of (9) by r new constraints (10) with linear left-hand sides.

Let us draw attention on the right-hand side of inequalities (10). The value of non-linear function $\min_{j=1,...,r} (w_j \cdot u_{jl})$ depends on the vector $w = (w_1,..., w_r)$, but it is always $w_q \cdot u_{q,l}$. Here q may be arbitrary from the set $\{1,..., r\}$. In order to avoid the non-linear function "min" in (10), we have to consider non-intersecting subsets of weights W^0 under condition that the function $\min_{j=1,..,r} (w_j \cdot u_{jl})$ has the same values for all weights w belonging to each subset. Let us divide the set of weight W^0 into subsets $W_1^0,..., W_r^0$ such that $W^0 = W_1^0 \cup,..., \cup W_r^0$ and for all $i \ne j, W_i^0 \cap W_j^0 = \varnothing$. Here W_q^0 is the set of vectors $w = (w_1,..., w_r)$ such that the following equality is valid:

$$\min_{j=1,..,r} w_j \cdot u_{j,l} = w_q \cdot u_{q,l} \cdot \qquad (11)$$

In other words, the function $\min_{j=1,...,r} (w_j \cdot u_{jl})$ has the same values $w_q \cdot u_{q,l}$ for all weights w from subsets W_q^0. It should be noted that W_q^0 may be empty. This implies that there are no vectors satisfying the above equality.

The next question is how to determine every subset W_q^0. The answer is very simple. The subset W_q^0 is produced by the following set of linear inequalities:

$$w_q \cdot u_{q,l} \le w_j \cdot u_{j,l} \ , \ j = 1,..., r. \qquad (12)$$

At that, the vector w belongs to the subset W_q^0 if it simultaneously satisfies all the above inequalities.

Now we return to optimization problems (5) and (7). By taking into account the above, we can replace problems (5) and (7) by sets of r linear programming problems of the following form:

$$L_k(q) = \inf_{w} \min_{j=1,...,r} (w_j \cdot u_{jk}), \qquad (13)$$

$$U_k(q) = \sup_{w} \min_{j=1,...,r} (w_j \cdot u_{jk}), \qquad (14)$$

subject to $w \in W$ and

$$w_t \cdot u_{t,i} \ge w_q \cdot u_{q,l}, \ t = 1,..., r, \qquad (15)$$

$$w_q \cdot u_{q,l} \le w_j \cdot u_{j,l}, \ j = 1,..., r. \qquad (16)$$

Here the first part of constraints ($w \in W$) is produced by initial judgments about weights. The second part of constraints (15) corresponds to the linear equivalent of the left-hand side of (9). The third part of constraints (16) corresponds to the linear equivalent of the right-hand side of (9) or to subset W_q^0.

Let us analyze constraints (15)-(16). One can observe that, by replacing the index **t** by the index **j** in (15), we have for every **j** two constraints

$$w_q \cdot u_{q,l} \le w_j \cdot u_{j,i},\ j = 1,...,r,$$

$$w_q \cdot u_{q,l} \le w_j \cdot u_{j,l},\ j = 1,...,r.$$

It is obvious that the above constraints can be replaced by one constraint

$$w_q \cdot u_{q,l} \le w_j \cdot \min\{u_{j,i}, u_{j,l}\},\ j = 1,...,r \quad (17)$$

This implies that linear programming problems (13)-(16) can be simplified.

Now we can apply the previously obtained results for solving the problems (13)-(16). It follows from (6) that the problem (13) can be rewritten now as

$$L_k(q) = \min_{i=1,...,M} \min_{j=1,...,r} (w_j^{(i)}(q) \cdot u_{j,k}) \quad (18)$$

Here $w^{(i)}(q) = (w_1^{(i)}(q),..., w_r^{(i)}(q))$ is the i-th extreme point of the polytope formed by constraints $w \in W$,(17).

It follows from (8) that the problem (14) can be rewritten now as

$$U_k(q) = \sup_w G \quad (19)$$

Subject to $w \in W,\ (17)$ and

$$G \le w_j \cdot u_{j,k},\ j = 1,...,r.$$

It is interested to note that constraints (17) do not depend on the considered DA with the number **k** and depend only on two comparable DA`s. Finally, the optimal values of L_k and U_k can be found as:

$$L_k = \min_{q=1,..,r} L_k(q), \quad (20)$$

$$U_k = \min_{q=1,...,r} U_k(q). \quad (21)$$

The above can be regarded as a proof to the following proposition.

Proposition 3: Suppose that there is a comparative judgment formalized as (9). Then the optimal values L_k and U_k in problems (5) and (7) can be found by means of (20) and (21), respectively. Here L_k (q) and U_k (q) are computed from (18) and (19), respectively.

Proposition 4: Suppose that there is a comparative judgment and there is no prior information about weights of criteria. Then $L_k = 0$ for all k = 1,…,n, and the optimal DA is determined only by comparing the upper bounds U_k.

Proof. If we do not have prior information about weights of criteria, then the set of weights W_q^0 by a fixed q is defined only by constraints (17) and by the set W^0. Let us rewrite (17) as follows:

$$w_q \le w_j \cdot \min\{u_{j,i}, u_{j,l}\} / u_{q,l}.$$

This implies that w_q is restricted only from above and there exist the extreme points of the form:

$$(1,..,0_q,..,0), (0,1,..,0_q,...,0),..,(0,...,0_q,..,1)$$

For all these points, we have w_q = 0. According to Corollary 1, we get $L_k(q) = 0$. Consequently, there holds

$L_k = 0$, as was to be proved.

Example 2: Let us return to example 1, first, we assume that there is no prior information about weight of criteria. The DM supposes that the first site is more preferable than the third one. This implies that there holds

$$\min_{j=1,..,r}(w_j \cdot u_{j1}) \ge \min_{j=1,...,r}(w_j \cdot u_{j,3})$$

For computing the lower bound $L_k(q), k = 1,...,4$ we have to computing the lower, we have to find a new set of extreme points by taking into account new constraints (17)

$$w_q \cdot u_{q,l} \le w_j \cdot \min\{u_{j,i}, u_{j,l}\},\ j = 1,...,3$$

For instance, these constraints by q = 1 and k =1 are of the form

$$w_1 \cdot u_{1,3} \le w_1 \cdot \min\{u_{1,1}, u_{1,3}\},$$

$$w_1 \cdot u_{1,3} \le w_2 \cdot \min\{u_{2,1}, u_{2,3}\},$$

$$w_1 \cdot u_{1,3} \le w_3 \cdot \min\{u_{3,1}, u_{3,3}\},$$

Or

$$w_1 \cdot 3 \le w_1 \cdot \min\{5, 3\},$$

$$w_1 \cdot 3 \le w_2 \cdot \min\{3, 5\},$$

$$w_1 \cdot 3 \le w_3 \cdot \min\{2, 3\}.$$

However, in accordance with proposition 4, we do not need to search the extreme points because there is no prior information about weight of criteria and

$$L_k = 0, k = 1,...,4.$$

For computing the upper bounds $U_k(q), k = 1,...,4$, we have to solve 12 linear programming problems. For instance, the problem by q =1 and k =1 is of the form

$$U_k(q) = \sup_w G$$

Subject to $w_1 + w_2 + w_3 = 1, w_i \ge 0, i = 1,...,3$ and

$$G \le w_1 \cdot 3,$$
$$G \le w_2 \cdot 3,$$
$$G \le w_3 \cdot 2,$$
$$w_1 \cdot 3 \le w_1 \cdot 3,$$
$$w_1 \cdot 3 \le w_2 \cdot 3,$$
$$w_1 \cdot 3 \le w_3 \cdot 2.$$

Hence, we obtain

$$U_1(1) = 30/31,\ U_2(1) = 4/9,\ U_3(1) = 6/7,\ U_4(1) = 2/7$$

and $U_k(q) = 0, k = 1,...,4,$ $q = 2, 3$ Then

$$U_1 = 30/31,\ U_2 = 4/9,\ U_3 = 6/7,\ U_4 = 2/7$$

Now we can conclude that hat the first site is optimal. By using the obtained upper bounds, we can also rank the sites as follows:

$$1 \succ 3 \succ 2 \succ 4.$$

If we use the initial information about weights of criteria given in Example 1, than a new system of constraints produces the empty sets of possible weights. This implies that the comparative judgment conflicts with this initial information.

Proposition 5: Suppose that the i-th DA is more preferable than the l-th DA. Let $u_{t,i}$ be the rate of the i-th DA with respect to the t-th criterion. Moreover, we suppose that

$$u_{t,i} < \min_{j=1,...,r} u_{j,l}.$$

Then the above comparative information is conflicting.

Proof. Assume q = t. According to (15), we can write

$$w_t \cdot u_{t,i} \ge w_t \cdot u_{t,l},\ t = 1,...,r$$

Hence $u_{t,i} \ge u_{j,l}$. We get a contradiction because $u_{t,i} < u_{j,l}$ due to the stated condition. Moreover, w_t cannot be 0 for all $t = 1,...,r$, because the sum of all weights is 1. This implies that the set W_q^0 is empty for all $q = 1,...,r$.

Proposition 5 allows us to determine the conflicting comparative judgments without solving the corresponding linear programming problems.

V. CONCLUSION

An approach to solving a pessimistic MCDM problem has been proposed in the paper. The stated problem has three main peculiarities. Firstly, it uses the non-linear maximin technique for aggregating the criteria. Secondly, the available information about weights of criteria is partial and this information produces a set of weights such that the lower and upper bounds for the global criterion have been introduced and used for comparing the DA`s. Lastly, the comparative judgments have been applied for reducing the possible set of the criteria weights. The comparative judgments lead to non-linear constraints and make the solved optimization problem rather complicated. At the same time, the main advantage of the proposed approach to solving the MCDM problem is that the non-linear problems are replaced by a finite set of standard and simple linear programming problems whose solution does not meet any difficulties.

It should be noted that the simplest case has been studied in the paper when only one comparative judgment is provided by the DM. However, the proposed approach can be extended on a more complicated case without difficulties. Moreover, only precise rates or weights of DA`s have been taken into account. Therefore, a direction for further work is to investigate the cases when we have only partial information about DA`s.

REFERENCES

[1]. M. Beynon. DS/AHP method: A mathematical analysis, including an understanding of uncertainty. European Journal of Operational Research, 140:148-164, 2002.

[2]. J.Y. Halpern and R. Fagin. Two views of belief: Belief as generalized probability and belief as evidence. Artificial Intelligence, 54:275-317, 1992.

[3]. K.S. Park and S.H. Kim. Tools for interactive multi-attribute decision making with incompletely identified information. European Journal of Operational Research, 98:111-123, 1997.

[4]. S.H. Kim and B.S. Ahn. Interactive group decision making procedure under incomplete information. European Journal of Operational Research, 116:498-507, 1999.

[5]. M. Beynon, B.Curry, and P. Morgan. The Dempster-Shafer theory of evidence. An alternative approach to multicriteria decision modelling. Omega, 28:37-50, 2000.

[6]. E.W.T. Ngai. Selection of web sites for online advertising using the AHP. Information and Management, 40(4):233 - 242, 2003.

[7]. K.-M. Osei-Bryson. Supporting knowledge elicitation and consensus building for Dempster-Shafer decision models. International Journal of Intelligent Systems, 18:129-148, 2003.

[8]. T.Tervonen, R. Lahdelma, and P. Salminen. A method for elicitating and combining group preferences for stochastic multicriteria acceptability analysis. TUCS Technical Report 638, Turku Centre for Computer Science, Turku, Finland, November 2004.

[9]. Noghin V.D. A Simplified Variant of the Analytic Hierarchy Processes Based on a Nonlinear Scalar zing Function. Computational Mathematics and Mathematical Physics, V. 44, No. 7, pp. 1194-1202, 2004.

[10]. L.V. Utkin and Th. Augustin. Decision making under incomplete data using the imprecise Dirichlet model. International Journal of Approximate Reasoning, 44(3):322-338, 2007.

[11]. J. Mustajoki, R.P. Hamalainen, and M.R.K. Lindstedt. Using intervals for global sensitivity and worst-case analyses in multiattribute value trees. European Journal of Operational Research, 174(1):278-292, 2006.

[12]. Nguyen Van Hieu, Lev V. Utkin, Dang Duy Thang. A pessimistic approach for solving a multi-criteria decision making. Proceeding of the Fourth International Conference on Knowledge and Systems Engineering, 4: 121-127, 2012.