STRESS ANALYSIS IN AN ELASTIC HALF SPACE DUE TO A THERMOELASTIC STRAIN

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Abstract:- The stress distribution on elastic space due to nuclei of thermo elastic strain distributed uniformly on the circumference of a circle of radius R situated in the place z= λ of the elastic semi space of Hookean model has been discussed by Nowacki: The Force stress and couple stress have been determined . The fore stress reduces to the one obtained by Nowacki for classical elasticity.

Key words: Stress distribution, semi space of Hookean model, force stress and couple stress.

Introduction:

Analysis of stress distribution in elastic space due to nuclei of thermoelastic strain distributed uniformly on the circumference of a circle of radius r situated in the plane Z = h of the elastic semi space of Hookean model has been discussed by Nowacki.

This note is an extension of the analysis of above problem for micropolar elastic semi-space. Force stress $^{\sigma}$ji and couple stress $^{\mu}$ji have been determined due to presence of nuclei of thermoelastic strain situated in the place Z = h inside the semi space. The force stress reduces to the one obtained by Nowascki for classical elasticity.

Basic Equations:

We consider a homogenous isotropic elastic material occupying the sami infinite region Z ≥ O in cylindrical polar coordinate system (r, θ, Z). It has been shown by Nowacki [64] that is in the case when the macrodisplacement vector $\vec{u}$ and microrotation $\vec{w}$ depend only on r and z the basic equations of equilibrium of micro-polar theory of elasticity are decomposed into two mutually independent sets. Here we shall be concerned with the set $\vec{u}$ = (u_r, O, u_z) and the rotation vector $\vec{w}$ = (O, ϕ_θ,O):

$$(\mu+\alpha)(\nabla^2-\frac{1}{{}_r2})u_r+(\lambda+\mu-\alpha)\frac{\partial e}{\partial r}-2\alpha\frac{\partial\phi_\theta}{\partial z}=\varsigma\frac{\partial T}{\partial r}$$

$$(\mu+\alpha)(\nabla^2_{u_z}-+(\lambda+\mu-\alpha)\frac{\partial e}{\partial r}+2\alpha.\frac{1}{r}\frac{\partial}{\partial r}(r\phi_\theta)=\varsigma\frac{\partial T}{\partial z}$$

$$(\gamma+\in)(\nabla^2-\frac{1}{{}_r2}\phi_\theta+2\alpha(\frac{\partial u_r}{\partial z}-\frac{\partial u_z}{\partial r})-4\alpha\phi_\theta=0$$

…..(6.1)

Where e = $\frac{1}{r}\frac{\partial}{\partial r}(r\mu_r)+\frac{\partial u_z}{\partial z}$

$\nabla^2 \equiv \partial_r^2+\frac{1}{r}\partial_r+\partial_z^2$

ζ= $(3\lambda + 2\mu)\,^{\alpha}t$

u_r, u_z = displacement components

ϕ_θ = Component of rotation vector

λ, μ, α, γ, ∈ = elastic constants

T (r, z) = temperature distribution

$^{\alpha}t$ = coefficient of thermal expansion.

To the displacement vector $\overrightarrow{u}$ (u_r , O, u_z) and the rotation vector $\overrightarrow{w}$ = (O, ϕ_θ, O) is ascribed the following state of force stress σ_{ij} and couple stress μ_{ij}

$$\sigma_{ij} = \begin{Vmatrix} \sigma_{rr} & 0 & \sigma_{rz} \\ 0 & \sigma & 0 \\ \sigma_{zr} & 0 & \sigma_{zz} \end{Vmatrix}$$

$$\mu_{ij} = \begin{Vmatrix} 0 & \mu_{r\theta} & 0 \\ \mu_{\theta r} & 0 & \mu_{\theta z} \\ 0 & \mu_{z\theta} & 0 \end{Vmatrix}$$

<u>Stress-Strain relations :</u>

The relation between stress tensor σ_{ij} , μ_{ij} and displacement $\overrightarrow{u}$ and rotation $\overrightarrow{w}$ in the cylindrical coordinates are given by

$$\sigma_{rr} = 2\mu\frac{\partial u_r}{\partial r} + \lambda e - T$$

$$\sigma_{\theta\theta} = 2\mu\frac{u_r}{r} + \lambda e - T$$

$$\sigma_{zz} = 2\mu\frac{\partial u_z}{\partial z} + \lambda e - T$$

$$\sigma_{rz} = \mu(\frac{\partial u_{zz}}{\partial r} + \frac{\partial u_r}{\partial z}) - \alpha(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}) + 2\alpha\phi_\theta$$

$$\sigma_{zr} = \mu(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}) + \alpha(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}) - 2\alpha\phi_\theta$$

$$\mu_{r\theta} = \gamma(\frac{\partial \phi_\theta}{\partial r} - \frac{\phi_\theta}{r}) + \in(\frac{\partial \phi_\theta}{\partial r} + \frac{\phi_\theta}{r})$$

$\mu_{\theta r} \quad = \quad \gamma(\frac{\partial\phi_\theta}{\partial r} - \frac{\phi_\theta}{r}) + \in (\frac{\partial\phi_\theta}{\partial r} + \frac{\phi_\theta}{r})$ ….(6.2)

$\mu_{\theta z} \quad = \quad (\gamma - \in) \frac{\partial\phi_\theta}{\partial z}$,

$\mu_{z\theta} \quad = \quad (\gamma - \in) \frac{\partial\phi_\theta}{\partial z}$

Following Nowacki [108], we introduce displacement potentials ϕ, Ψ and rotation potential V such that

$\mu_r \quad = \quad \frac{\partial\phi}{\partial r} + \frac{\partial^2\psi}{\partial_r \partial z}$

$\mu_z \quad = \quad \frac{\partial\phi}{\partial z} - \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial\psi}{\partial r})$ …. (6.3)

$\phi_\theta \quad = \quad \frac{\partial v}{\partial r}$

Substituting (6.3) in (6.2) we get

$$(\lambda + 2\mu)\frac{\partial}{\partial r}(\nabla^2\theta) + \frac{\partial^2}{\partial z \partial r}\left[(\mu+\alpha)\nabla^2\psi - 2\alpha v\right] \ \Big| = \varsigma \frac{\partial T}{\partial r} \quad ……(6.4)$$

$$(\lambda + 2\mu)\frac{\partial}{\partial r}(\nabla^2\theta) - (\nabla^2 -)\frac{\partial^2}{\partial z^2}\left[(\mu+\alpha)\nabla^2\psi - 2\alpha v\right] = \varsigma \frac{\partial T}{\partial r}$$

$$\frac{\partial}{\partial r}\left[(\gamma + \in)\nabla^2 - 4\alpha\right] v + 2\alpha = \frac{\partial}{\partial r}\nabla 2\Psi = 0$$

The above equations are satisfied if

$\nabla^2 \nabla^2 \ \phi \ = \ m \nabla^2 T$

$\nabla^2 ((^2\nabla^2 -1) V = 0$ ….(6.5)

Where $£^2 = \frac{(\mu+\alpha)\gamma + \in)}{4\alpha\mu}$, m $= \frac{\varsigma}{\lambda + 2\mu}$, and V and Ψ are related by

$\nabla^2\Psi \ = -2 \left[(\frac{\gamma + \in}{4\alpha})\nabla^2 - 1 \right] \quad V$ …. (6.6)

To solve (6.5) we write

$= \quad \phi' \quad + \quad \phi''$ ….. (6.7)

$V = V' + V''$

Where ϕ' and V' are particular integrals for non-homogeneous part and ϕ'', V'' are general solutions of homogeneous part. Now for particular integral we have

$\nabla^2 \quad \phi' \quad = \quad mT$ … (6.8)

and $\quad V' \quad = \quad 0$

and for general solution we have

$$\nabla^2 \quad \nabla^2 \quad \phi'' \quad = \quad 0$$

$\nabla^2 (\ell^2 \nabla^2 - 1) \quad V'' \quad = \quad 0$ … (6.9)

<u>Solution of the title problem :</u>

We consider nuclei of thermo elastic strain distributed uniformly on the circumference of a circle of radius r and situated in the plane z = h inside the elastic half space. The stress distribution σ_{ij} can be considered as sum of two stress systems $\left|\bar{S}\right|$ and $\left|\bar{\bar{S}}\right|$. The system $\left|\bar{S}\right|$ constitute stress distribution σ_{ij} of infinite elastic space containing two nuclei of thermoelastic strains situated in the planes z = h and z = -h distributed uniformly along the circumferences of the circles, each of radius r. The second system $\left|\bar{\bar{S}}\right|$ constitutes stress distribution σ_{ij} corresponding to elastic semi-space in the isothermal state. The stress σ''_{ij} is so chosen that the boundary conditions on the plane z = O.

$$\sigma_{zz} = 0, \quad \sigma_{zr} = 0, \quad \mu_{z\theta} = 0$$

are satisfied.

The thermoelastic displacement potential ϕ' corresponding to σ_{ij} satisfies the equation

$\nabla^2\phi \quad = \quad m\delta(R^r - R)[\delta(z-h) - \delta(z+h)]$ ….(6.10)

Where $r^2 = x^2 + y^2$ and $\delta(x)$ represents Dirac – delta function.

Representing the right hand side of the equations (6.10) by the Fourier Integral

$m\delta(r-R)[\delta(z-h) - \delta(z+h]$

$$= \frac{mR}{\pi} \int^{\infty} \int_{\xi}^{\infty} J_o(\xi r) J_o(\xi R) \left[Cosr(z-h) - Cosr(z+h) \right]$$

The solution of (6.10) is represented by the integral

$$\phi' = \frac{mR}{\pi} \int_{o}^{\infty} J_o(\xi R) J_o(\xi r) \left[e^{-} \xi(z-h) -_{e}^{-} \xi(z-h) \right] d\xi, |z| - h > 0$$

…..6.11)

$$= \frac{mR}{\pi} \int_o^\infty J_o(\xi R) J_o(\xi r) \left[e^- \xi(z-h) -_e^- \xi(z-h) \right] d\xi, |z| - h > 0$$

…..6.12)

The stress distribution for the system ($\bar{S}$) is obtained

$$\sigma_{rr} \quad = \quad 2\mu \left[\left(\frac{\partial^2 \phi'}{\partial r^2}\right) - \nabla^2 \phi' \right]$$

$$= \quad m\mu R \int_o^\infty \xi^2 J_o(\xi R) \left[J_o(\xi R) + \frac{1}{\xi r} J_1(\xi r) \right] \left[e^{(\xi(z-h)} - e^{-\xi(z+h)} \right] d\xi$$

$$\sigma\theta\theta' \quad = \quad 2\mu \left(\frac{1}{r}\frac{\partial \phi'}{\phi r} - \nabla^2 \phi'\right) \quad = -2\mu \left(\frac{\partial^2 \phi'}{\partial r^2} + \frac{\partial^2 \phi'}{\partial z^2}\right)$$

$$= \quad m\mu R \int_o^\infty \xi^2 J_o(\xi R) \left[J_o(\xi r) + J_o''(\xi r) \right] \left[e^{(\xi(z-h)} - e^{-\xi(z+h)} \right] d\xi$$

General Solution for Homogeneous Equations:

Applying Kankel transform to equation (6.9), the general solution for half space is given by

$$\phi'' \quad = \int_o^\infty \xi(A + B\xi z) e^{-\xi z} Jo(\xi r) d\xi \qquad \text{… (6.14)}$$

$$\text{and} \quad V'' \quad = \int_o^\infty \xi(L_e^{-\xi z} + Me^{-\sigma z}) J_o(\xi r) d\xi \qquad \text{…. (6.15)}$$

where $\sigma^2 = \xi^2 + \frac{1}{\ell^2}$ and L,M,A, B are some functions of ξ, to be determined by boundary conditions.

Equations (6.4) give

$$L \; = \; - \frac{\lambda + 2\mu}{\mu} \xi B. \qquad \text{…(6.16)}$$

Knowing the functions ϕ" , Ψ" and V" the force stresses and couple stresses are calculated by the relations

$$\sigma''_{rr} \quad = \quad 2\mu \quad \frac{\partial u_r}{\partial r} + \lambda e = 2\mu \frac{\partial^2}{\partial r^2} (\phi'' + \frac{\partial \psi''}{\partial z}) + \lambda \nabla^2 \phi''$$

$\sigma''_{\theta\theta}$ = 2μ $\frac{1}{r}\frac{\partial}{\partial r}(\phi''+\frac{\partial \psi''}{\partial z})+\lambda\nabla^2\phi''$

σ''_{zz} = 2μ $\frac{\partial}{r}\frac{\partial}{\partial r}\left[\frac{\partial \phi''}{\partial z}-(\nabla^2-\frac{\partial^2}{\partial z^2})\psi''\right]+\lambda\nabla^2\phi''$

σ''_{zz} = $\frac{\partial}{\partial r}\left[\mu\left\{2\frac{\partial \phi''}{\partial z}-(\nabla^2-2\frac{\partial^2 z}{\partial z^2})\psi''\right\}\right.$

$+\left.\alpha\nabla^2\psi''-2\alpha V''\right]$

$\mu''_{r\theta}$ = $(\gamma\text{-}\in)$ $\frac{\partial^2 V''}{\partial_r^{\ 2}}$ - $(\gamma\text{-}\in)$ $\frac{1}{r}$ $\frac{\partial V''}{\partial_r}$

$\mu''_{\theta r}$ = $(\gamma\text{-}\in)$ $\frac{\partial^2 V''}{\partial_r^{\ 2}}$ - $(\gamma\text{-}\in)$ $\frac{1}{r}$ $\frac{\partial V''}{\partial_r}$

$\mu''_{z\theta}$ = $(\gamma\text{-}\in)$ $\frac{\partial^2 V''}{\partial r\partial z}$

$\mu''_{\theta z}$ = $(\gamma\text{-}\in)$ $\frac{\partial^2 V''}{\partial r\partial z}$

Since the bounding surface z = 0 is free from tractions, we have on z = O, $|S| + |\overset{=}{S}|$ = O

Thus

σ_{zz}	=	σ'_{zz}	+	σ''_{zz}	=	O
σ_{zr}	=	σ'_{zr}	+	σ''_{zr}	=	O
$\mu_{z\theta}$	=	$\mu'_{z\theta}$	+	$\mu''_{z\theta}$	=	O

Since $\mu'_{z\theta}$ = O, we get $\mu'_{z\theta}$ = O from $(6.18)_3$

This gives L = - M $\frac{\sigma}{\xi}$ … (6.19)

Also, from (6.16) we get

$$L = -L\frac{\sigma}{\xi} = (\frac{\lambda+2\mu}{\mu})(\frac{\xi^2}{\sigma})B$$

The solution of equation

$$\nabla^2\psi'' = -\frac{1}{2\alpha}\left[(\gamma+\in)\nabla^2 - 4\alpha\right]V''$$

Is obtained as

$$\psi'' = \frac{\lambda+\mu}{\mu}\int_o^\infty B\left(\frac{\lambda+2\mu}{\lambda+\mu}\xi z e^{-\xi z} + 2a_o\frac{\xi 3}{\sigma}e_-^{-\sigma z}\right)J_o(\xi r)d\xi$$

Where a_O $\frac{(\lambda+\in)(\lambda+2\mu}{4\mu(\lambda+\mu)}$

Boundary conditions (6.18) 1, 2 yield

A = 4 ao $\xi^2 P(\xi)$

B = $\frac{(2\mu}{(\lambda+\mu)}$ P (ξ) … (6.20)

Where P (ξ) = $\frac{mR\xi J_o(\xi R)e^{-\xi h}}{1+2a_o\xi 2(1-\xi/\sigma)}$

Substituting expressions for ϕ", Ψ" and V" with values of A and B in (6.20), we obtain σ_{ij} and μ_{ij} with the help of the relations (6.17)

$$\sigma''_{zz} = 3\mu\int_o^\infty\left[4a_0\xi^2 - \frac{2\mu}{\lambda+\mu}(2-\xi z\right]P(\xi)\xi^3 e -^{-\xi z}\ J_o(\xi r)d\xi$$

$$+2\mu\int_o^\infty\left[(1+\frac{\mu}{\lambda+\mu})(1-\xi z)e^{-\xi z} --2a_o\xi^2 e^{-\xi z}\right]_\xi 3P(\xi)J_o(\xi r)d\xi$$

$$\frac{4\mu}{\lambda+\mu}\int_o^\infty\xi^3 e^{-\xi zP}(\xi)J_o(\xi r)d\xi$$

$$\sigma''_{zr} = 2\mu\int_o^\infty\left[\frac{2\mu}{\lambda+\mu}(1-\xi z) - 4a_o\xi^2\right]P(\xi)e^{-\xi z}J_o'(\xi r)d\xi$$

$$+4(\mu-\alpha)\int_o^\infty\left[1+\frac{\mu}{\lambda+\mu}e^{-\xi z} + a_o\xi^3(1/\sigma-\sigma)e^{-\sigma z}\right]P(\xi)\xi^3 J_o'(\xi r)d(\xi)$$

$$+4\mu\int_o^\infty\left[1+\frac{\mu}{\lambda+\mu}(\xi z-2)e^{-\xi z} + 2a_o\xi\sigma^{-\sigma z}\right]\xi^3 P(\xi)J_o'(\xi r)d\xi$$

$$+\frac{4\alpha}{\lambda}\frac{(\lambda+2\mu}{+\mu}\int_o^\infty\xi^3(e^{-\xi z} - \frac{\xi}{\sigma}e^{-\sigma z})P(\xi)J_o'(\xi r)d\xi$$

$$\mu''_{r\theta} = \frac{-2(\lambda+2\mu)}{\lambda+\mu}\int_o^\infty(e^{-\xi z} - \frac{\xi}{\sigma}e^{-\sigma z})\left[(\lambda+\in)J_o''(\xi r) - (\lambda-\in).\frac{1}{r}J_o'(\xi r)\right]x\xi^3 P(\xi)$$

$$\mu''_{z\theta} = \frac{2(\lambda+\in)(\lambda+2\in)}{\lambda+\mu}\int_o^\infty (e^{-\xi z} - e^{-\sigma z})\xi^4 P(\xi) J'_o(\xi r) d\xi \qquad \text{…. (6.21)}$$

Stress distribution in the elastic half space is obtained by adding (6.13) and (6.21)

Thus

$$\sigma_{rr} = \sigma'_{rr} + \sigma_r{}''$$

$$= m\mu R \int_o^\infty \left[J_o(\xi r) + \frac{1}{\xi r} J_1(\xi r) \right] \left[e^{\xi(z-h)} - e^{-\xi(z-h)} \right] \xi^2 J_o(\xi R) d\xi$$

$$+2\mu \int_o^\infty \left[4a_o \xi^2 + \frac{2\mu}{\lambda+\mu} \xi z P \right] \left[\frac{1}{\xi r} J_1(\xi r) - J_o(\xi r) \right] \xi^3 P(\xi) e^{-\xi z} d\xi r$$

$$+4\mu \int_o^\infty \left[(1+\frac{\mu}{\lambda+\mu})(1-\xi z)e^{-\xi z} - 2a_o \xi^2 e^{-\sigma z} \right] \left[\frac{1}{\xi r} J_1(\xi r) - J_o(\xi r) \right] \xi^3 P(\xi) d\xi$$

$$\sigma_{\theta\theta} = \sigma_{\theta\theta}{}' + \sigma_{\theta\theta}{}''$$

$$= m\mu R \int_o^\infty \left[e^{\xi(z-h)} - e^{-\xi(z+h)} \right] \frac{1}{\xi r} . J_1(\xi r) J_o(\xi r) d\xi$$

$$+2\mu \int_o^\infty \left[4a_o \xi^2 + \frac{2\mu}{\lambda+\mu} \xi z \right] \frac{\xi^2}{r} P(\xi) e^{-\xi z} J_1(\xi r) d\xi$$

$$-4\mu \int_o^\infty \left[(1+\frac{\mu}{\lambda+\mu})(1-\xi z)e^{-\xi z} - 2ao\xi^2 e^{-\sigma z} \right] \frac{\xi 2}{r} P(\xi) J_1(\xi r) d\xi$$

$$-\frac{4\lambda\mu}{\lambda+\mu} \int_o^\infty \xi^3 e^{-\xi z} P(\xi) J_o(\xi r) d\xi$$

$$\sigma_{zz} = \sigma'_{zz} + \sigma_{zz}{}''$$

$$= -m\mu R \int_o^\infty \left[e^{-\xi(z-h)} - e^{-\xi(z+h)} \right] \xi^2 J_o(\xi R) J_1(\xi r) d\xi$$

$$+2\mu \int_o^\infty \left[4a_o \xi^2 - \frac{2\mu}{\lambda+\mu}(2-\xi z) \right] \xi^3 e^{-\xi z} P(\xi) J_o(\xi r) d\xi$$

$$+2\mu \int_o^\infty \left[(1+\frac{\mu}{\lambda+\mu})(1-\xi z)e^{-\xi z} - 2a_o \xi^2 e^{-\sigma z} \right] \xi^3 p(\xi) J_o(\xi r) d\xi$$

$$-\frac{4\lambda\mu}{\lambda+\mu}\int_o^\infty \xi^3 e^{-\xi z}P(\xi)J_o(\xi r)d\xi$$

$$\sigma_{zr}=\sigma_{zr}{}'+\sigma_{zr}{}''$$

$$=-m\mu R\int_o^\infty \xi^2\left[e^{\xi(z-h)}-e^{-\xi(z+h)}\right]J_o(\xi R)J_1(\xi r)d\xi$$

$$-2\mu\int_o^\infty\left[\frac{2\mu}{\lambda+\mu}(1-\xi z)-4a_o\xi^2\right]\xi^3P(\xi)e^{-\xi z}J_1(\xi r)d\xi$$

$$-4(\mu-\alpha)\int_o^\infty\left[(1+\frac{\mu}{\lambda+\mu})e^{-\xi z}+a_o(\frac{1}{\sigma}\sigma)\xi^3e^{-\sigma z}\right]\xi^3P(\xi)J_1(\xi r)d\xi$$

$$-4\mu\int_o^\infty\left[(1+\frac{\mu}{\lambda+\mu})(\xi z-2)e^{-\xi z}+2a_o\xi\sigma e^{-\sigma z}\right]\xi^3P(\xi)J_1(\xi r)d\xi$$

$$-\frac{4\alpha(\lambda+2\mu)}{\lambda+\mu}\int_o^\infty\left[(e^{-\xi z}-\frac{\xi}{\sigma}e^{-\sigma z})\xi^3P(\xi)J_1(\xi r)d\xi\right]$$

$$\mu_{r\theta}=\mu''_{r\theta}=-\frac{2(\lambda+2\mu)}{\lambda+\mu}\int_\circ^\infty\left[(e^{-\xi z}-\frac{\xi}{\sigma}e^{-\sigma z})\right]\left[\xi rJ_2(\xi r)-\in\xi J_o(\xi r)\right]xP(\xi)d\xi$$

$$\mu_{z\theta}=\mu_{z\theta}''=\frac{-2(\gamma+\in)(\lambda+2\mu)}{\lambda+\mu}\int_\circ^\infty(e^{-\xi z}-e^{-\sigma z})\xi^4P(\xi)J_1(\xi r)d\xi$$

$$\sigma_{rr}-\sigma_{\theta\theta}=(\sigma'_{rr}+\sigma_{rr}'')-(\sigma'_{\theta\theta}+\sigma_{\theta\theta}'')$$

$$=(\sigma'_{rr}-\sigma_{\theta\theta}')+\sigma''_{rr}-\sigma_{\theta\theta}'')$$

$$=-m\mu R\int_o^\infty\left[e^{\xi(z-h)}-e^{-\xi(z+h)}\right]\xi^2J_o(\xi R)J_2(\xi r)d\xi$$

$$+2\mu R\int_o^\infty\left[(4a_o\xi^2+\frac{2\mu}{\lambda+\mu}\xi z)e^{-\xi z}+2(1+\frac{\mu}{\lambda+\mu})(1-\xi z)e^{-\xi z}-2a_o\xi^2e^{-\sigma z}\right]3_P(\xi)$$

$$xJ_2(\xi r)d\xi\ldots \quad \ldots.(6.22)$$

For α = O, the micropolar couple stress vanishes and in that case $\gamma=\in$O, a_o= O, σ =. Thus we get from (6.22)

$$\sigma_{rr} = muR\int_o^\infty \left[J_o(\xi r) + \frac{1}{\xi r} \right] \left[e^{\xi(z-h)} - e^{-\xi(z+h)} \right] J_o(\xi r) d\xi$$

$$+ \frac{2\mu}{1-2} \int_\circ^\infty P(\xi) e^{-\xi z} \left[(2-\xi z) J_o(\xi r) + (2-\xi z) \frac{J_1(\xi r)}{\xi r} \right] \xi^3 d\xi$$

$$\sigma_{\theta\theta} = muR\int_o^\infty \left[J_o(\xi r) + \frac{1}{\xi r} J_1(\xi r) \right] \left[e^{\xi(z-h)} - e^{-\xi(z+h)} \right] \xi^2 J_o(\xi R) d\xi$$

$$+ \frac{2\mu}{1-2\varsigma} \int_\circ^\infty \left[(2\varsigma J_o(\xi r) - (2\varsigma - 2\xi z) \frac{J_1(\xi r)}{\xi r} \right] P(\xi \ ) e^{-\xi z} \xi^3 d\xi$$

$$\sigma_{zz} = muR\int_o^\infty \left[e^{\xi(z-h)} - e^{-\xi(z+h)} \right] \xi^2 J_o(\xi R) J_1(\xi r) d\xi$$

$$+ \frac{2\mu}{1-2\varsigma} \int_\circ^\infty P(\xi) \xi^4 e^{-\xi z} J_o(\xi r) d\xi$$

σ_{zr} = σ_{rz} = - $$-muR\int_o^\infty \left[e^{\xi(z-h)} - e^{-\xi(z+h)} \right] \xi^2 J_o(\xi R) J_1(\xi r) d\xi$$

$$-\frac{2\mu}{1-2\varsigma} \int_\circ^\infty (1-\xi z) P(\xi)^3 e^{-\xi z} J_1(\xi r) d\xi$$

….. (6.23)

urθ = uθr = 0

where P (ξ) reduces to (1-2ϑ) e^{-h} $J_o(\xi R)$. 1

Results in (6,23) have been obtained in for Hookean thermo elasticity.

References

Palov, N.A. Fundamental equations of the theory of asymmetric elasticity (in Russian). Prikil. Mat. Mekh. 28 (1964). 401.

Peterson, M.E., Rev. Geophys. 11, 355, 1973.

Paria, G., Advances in Applied Mechanics, Vol. 10, p. 73, Academic Press, New York (1967).

Puri, P. Int. J. Engng. Sci. 11, 735 (1973).

Paria, G.and Wilson, Proc. Camb. Phil. Soc. 58, 527 (1972).

Papadepolos, M. (1963) : The elastodynamics of moving loads, Jt. Australian Maths. Soc. 3., 79-92.

Robin, P.Y.F., AMER, Mineral. 59, 1286, 1974.

Rayleigh Lord, Proc. Lond. Math. Soc. 20, 225 (1888).

Roy Choudhuri and Loknath Debnath, Magneto thermo-elasticity plane waves in Rotating Media, Int. J. Engg. Sci. Vol. 21, No. 2, pp. 155-169, (1983).

Schaefer, H. : Versuch einer Elastizitatetheorie des sweidimensionalen Cosserat – Kontinuum, Miss. Angew. Math. Festschrift Tollmien, Berlin, 1962, Akademia Verlag.

Now ack. W.: Couple- Stresses in the Theory of Thermo-elasticity. Proc. Of the IUTPM symposis Vienna June 22-28 1966. Springer-verlag vier 1968.

Nay feh. A. and Nemat Nasser. J. Appl. Mech 39, Trans. ASME. 94, Ser. E, 108 (1972)

Purc. P. Int. J. Engng. Sci- 11.735 (1973).

Robin Py. F, AMER, MINERAL, 59, 1286, 1974.

Roy Choudhuri and Loknath Deba nath, Magneto Thermo- elasticity plane waves in Rotating Media, int. J- Engg. Sci. Vol., 21. No. 2 PP 155-169 (1983).

Wilkenson J.P.D. J. accounst. Soc. Am 49, 2002

Erigen A.C. Proc 9th Midwestern Mechanics cong. Part I wiley 2006.

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