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Incomplete Multigranulation Based on Rough Intuitionistic Fuzzy Sets

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***Abstract*— Rough set theory introduced by Pawlak [8] is based on equivalence relations. The definition of basic rough sets depends upon a single equivalence relation defined on the universe or several equivalence relations taken one each taken at a time. In the view of granular computing, classical rough set theory is based upon single granulation. The basic rough set model was extended to rough set model based on multi-granulations (MGRS) in [10], where the set approximations are defined by using multi-equivalences on the universe and their properties were investigated. Using hybridization of fuzzy set [20] with rough set the concept of fuzzy rough set was introduced by Dubois and Prade [2]. Recently, Fuzzy Rough Set Model Based on Multi-Granulation is introduced and studied by Wu and Kou [18]. Topological properties of rough sets introduced by Pawlak in terms of their types were recently studied by Tripathy and Mitra [16]. These were extended to the context of incomplete multi granulation by Tripathy and Raghavan [17]. In this paper we introduce incomplete multigranulation on intuitionistic fuzzy rough sets, study their basic properties and extend the topological properties in [17] to this context. Our findings are true for both complete and incomplete intuitionistic fuzzy rough set models based upon multi granulation.**

***Keywords*— Rough Sets, Fuzzy rough sets, Intuitionistic Fuzzy Rough Sets, type of rough sets, multi granular fuzzy rough sets and multigranular intuitionistic fuzzy rough sets.**

I. Introduction

Two of the most significant models which have been developed to enhance the modeling capability of basic sets are the notion of fuzzy sets introduced by L.A.Zadeh [19] and the notion of rough set introduced by Pawlak [8, 9]. However, the relative scarcity of equivalence relations led to the development of several extensions of this basic notion. One such extension is the rough sets based upon tolerance relations instead of equivalence relations. These rough sets are sometimes called incomplete rough set models. In the view of granular computing, classical rough set theory is researched by a single granulation. The basic rough set model has been extended to rough set model based on multi-granulations (MGRS) in [10], where the set approximations are defined by using multiple equivalence relations on a universe. Using similar concepts, that is taking multiple tolerance relations instead of multiple equivalence relations; incomplete rough set model based on multi-granulations was introduced in [11]. Several fundamental properties of these types of rough sets have been studied [10, 11, 13].The concept in [10] has been extended to rough fuzzy set model based on multi-granulations by Wu and Kou [19]. Very recently, this concept was generalised to incomplete multi granulation based on rough fuzzy sets [18]. Intuitionistic fuzzy set introduced by Attanasov [1] is a generalised notion of fuzzy sets. In this paper, we introduce both complete and incomplete intuitionistic fuzzy rough set models based on multi-granulations.

Employing the notions of lower and upper approximations of rough sets, an interesting characterization of rough sets was made by Pawlak in [9], where he introduced the types (originally called kinds) of rough sets. There are two different ways of characterising rough sets; the accuracy coefficient and the topological characterisation introduced through the notion of types. As mentioned by Pawlak [9], for fruitful applications of rough sets we need to combine both types of information; the accuracy measure as well as the topological classification of the rough set under consideration. Keeping this in mind, Tripathy and Mitra [16] studied the types of rough sets by finding out the types of union and intersection of rough sets of different types. In [17], some of these topological properties of rough sets were extended to MGRS on incomplete information systems. Also, similar kind of study is done by Tripathy and Nagaraju [18] for MGRFS. In this paper we introduce the notion of MGIFRS on complete and incomplete information systems and study similar properties.

II. Definitions And Notations

In this section we put forth several notations and define several concepts, which shall be used by us in presenting our work. In the next subsection we introduce the basic rough sets as introduced by Pawlak [8].

A. *Basic Rough Sets*

Let U be a universe of discourse and R be an equivalence relation over U. By U/R we denote the family of all equivalence class of R, referred to as categories or concepts of R and the equivalence class of an element x ϵ U, is denoted by $[x]_R$. By a knowledge base, we understand a relational system K = (U, P), where U is as above and P is a family of equivalence relation over U. For any subset Q($\neq \phi$) $\subseteq$ P, the intersection of all equivalence relations in Q is denoted by IND(Q) and is called the indiscernibility relation over Q. Given any X ⊆ U and R ϵ IND (K), we associate two subsets,

$\underline{R}X = \bigcup\{Y \in UR \subseteq Y\}$ and $\bar{R}X = \bigcup\{Y \in U \mid R : Y \bigcap X \neq \emptyset\}$ called the R-lower and R-upper approximations of X respectively. The R-boundary of X is denoted by BN_R (X) and is given by $BN_R(X) = \bar{R}X - \underline{R}X$. The elements of $\underline{R}X$ are those elements of U, which can certainly be classified as elements of X, and the elements of $\bar{R}X$ are those elements of U, which can possibly be classified as elements of X, employing knowledge of R. We say that X is rough with respect to R if and only if $\bar{R}X \neq \underline{R}X$, equivalently $BN_R(X) \neq \phi$. X is said to be R-definable if and only if $\bar{R}X = \underline{R}X$ or $BN_R(X) = \phi$.

B. Multigranular Rough Sets

In the view of granular computing (proposed by L. A. Zadeh), an equivalence relation on the universe can be regarded as a granulation, and a partition on the universe can be regarded as a granulation space [5, 6]. Several measures in knowledge base closely associated with granular computing, such as knowledge granulation, granulation measure, information entropy and rough entropy, were discussed in [5, 6, 7]. Qian and Liang brought forward a rough set model based on multi-granulations [10], which is established by using multi equivalence relations. A rough set based upon multi-granulation is defined as follows [10]:

Definition B.1: Let K= (U, **R**) be a knowledge base, **R** be a family of equivalence relations, X ⊆ U and P,Q ϵ **R.** We define the lower approximation and upper approximation of X in U as

(B.1) $\underline{P+Q}(X) = \{x \in U / [x]_P \subseteq X or [x]_Q \subseteq X\}$ and

(B.2) $\overline{P+Q}(X) = (\underline{P+Q}(X^C))^C$

Several properties of such type of rough sets were established in [67]. However, some of these proofs were found to be faulty by Wu and Kou [82].

C. Fuzzy Sets and Rough Fuzzy Sets

The first model to capture uncertainty in data is fuzzy set. We define it in the next subsection.

1) Fuzzy Sets: As mentioned in the introduction, fuzzy set introduced by Zadeh [20] is one of the approaches to capture vagueness in concepts. In the fuzzy set approach every member x of a set X ⊆U is associated with a grade of membership, which we denote by X(x) and is a real number lying in [0, 1]. The set of all functions from U to the unit interval [0, 1], is called the fuzzy power set of U and is denoted by F(U). It follows that P(U)⊆F(U).

2 Rough Fuzzy Sets: In the beginning when rough sets were introduced by Pawlak in the early 1980s, these two theories were supposed to be rival theories. But it was established by Dubois and Prade [2] that instead of being rival theories, these two theories complement each other. In fact they combined these two concepts to develop the hybrid models of fuzzy rough sets and rough fuzzy sets.

The notion of rough fuzzy sets was introduced by Dubois and Prade [2] as follows.

D. Rough Fuzzy Sets Model Based on Multi-Granulations

The concept of multi-granular rough sets was extended to define fuzzy rough sets based on multigranulation by Wu and Kou [107] as follows:

Definition D.1: Let K = (U, R) be a knowledge base, R be a family of equivalence relations on U and P, Q ϵ R. For ∀*X* ϵ *F*(*U*), the lower approximation $\underline{P+Q}(X)$ and upper approximation $\overline{P+Q}(X)$ of X based equivalence relations P, Q are defined as follows:

(D.1) ∀x ϵ U, $\underline{P+Q}(X)$ (x) = $\inf_{y \in [x]_P} X(y) \vee \inf_{y \in [x]_Q} X(y)$,

(D.2) ∀x ϵ *U*, $\overline{P+Q}(X)$ (x) = $((P+Q)(X^C))^C(x)$.

If $\underline{P+Q}(X)$ = $\overline{P+Q}(X)$ then X is called definable, otherwise X is called a fuzzy rough set with respect to multigranulations P and Q. The pair $((\underline{P+Q})(X), (\overline{P+Q})(X))$ is called a MG-fuzzy rough set on multi-granulations P and Q. It has been illustrated in [82] that fuzzy rough sets based on multi-granulations and fuzzy rough sets based on single granulations are different. The following properties of MG-fuzzy rough sets on multi-granulations were
established in [107].

Property D.1: Let K= (U, **R**) be a knowledge base, **R** be a family of equivalence relations. For every X ϵ F(U) and P,QϵR , the following properties hold true.

(D.3) $\underline{P+Q}(X) \subseteq X \subseteq \overline{P+Q}(X)$

(D.4) $\underline{P+Q}(\phi) = \overline{P+Q}(\phi) = \phi$

(D.5) $\underline{P+Q}(U) = \overline{P+Q}(U) = U$

(D.6) $\underline{P+Q}(X^C) = (\overline{P+Q}(X))^C$

(D.7) $\underline{P+Q}(X) = \underline{P}(X) \cup \underline{Q}(X)$

(D.8) $\overline{P+Q}(X) = \overline{P}(X) \cap \overline{Q}(X)$

(D.9) $(\underline{P+Q})((\underline{P+Q})(X)) = \underline{P+Q}(X) \subseteq (\overline{P+Q})(\underline{P+Q})(X)$

(D.10) $(\underline{P+Q})((\overline{P+Q})(X)) \subseteq \overline{P+Q}(X) = (\overline{P+Q})(\overline{P+Q})(X)$

(D.11) $(\underline{P+Q})(X) = \underline{Q+P}(X)$, $(\overline{P+Q})(X) = (\overline{Q+P})(X)$

Property D.2: Let K= (U, **R**) be a knowledge base, **R** be a family of equivalence relations. For every X, Y F(U) and P,Q R ,the following properties hold true

(D.12) $\underline{P+Q}(X \cap Y) \subseteq \underline{P+Q}(X) \cap \underline{P+Q}(Y)$

(D.13) $\underline{P+Q}(X \cup Y) \supseteq \underline{P+Q}(X) \cup \underline{P+Q}(Y)$

(D.14) $\overline{P+Q}(X \cap Y) \subseteq \overline{P+Q}(X) \cap \overline{P+Q}(Y)$

(D.15) $\overline{P+Q}(X \cup Y) \supseteq \overline{P+Q}(X) \cup \overline{P+Q}(Y)$

III. Intuitionistic Fuzzy Sets and Rough Intutionistic Fuzzy Sets

The notion of intuitionistic fuzzy sets was introduced by Attanasov [3] as an extension of the notion of fuzzy sets. It has been found to be more realistic as the nonmembership function is independent of the membership function but both are related to each other through a constraint. In fact, we have the following definition.

A. *Intuitionistic Fuzzy Sets*

In the intuitionistic fuzzy set approach every member x of a set $X \subseteq U$ is associated with a grade of membership and a grade of nonmembership, which we denote by MX(x) and NX(x) respectively. Both MX(x) and NX(x) are real number lying in [0, 1], such that $0 \le MX(x) + NX(x) \le 1$ for all x ϵ U. The set of all functions from U to J, where J = {(m, n) | m, n ϵ [0, 1] and $0 \le m + n \le 1$}, is called the intuitionistic fuzzy power set of U and is denoted by IF(U). It follows that P(U) ⊆ F(U) ⊆ IF(U).

The function HX(x) = 1- (MX(x) +NX(x)) for all x ϵ U is called the hesitation function for X. It is easy to see that for a fuzzy set X, NX(x) = 1 – MX(x) and HX(x) = 0, for all x ϵ U.

For X, Y ϵ U, some operations on intuitionistic fuzzy sets are defined as follows:

(A.1) $M(X \cap Y)(x) = \min\{MX(x), MY(x)\}$ and $N(X \cap Y)(x) = \max\{NX(x), NY(x)\}$, for all x ϵ U;

(A.2) $M(X \cup Y)(x) = \max\{MX(x), MY(x)\}$ and $N(X \cup Y)(x) = \min\{NX(x), NY(x)\}$, for all x ϵ U;

(A.3) $(MX^c)(x) = NX(x)$ and $(NX^c)(x) = MX(x)$ for all xϵU;

(A.4) $X \subseteq Y$ iff $MX(x) \le MY(x)$ and $NX(x) \ge NY(x)$, for all x ϵ U

(A.5) $M(X\backslash Y)(x) = \min\{MX(x), NY(x)\}$ and $N(X\backslash Y)(x) = \max\{NX(x), MY(x)\}$, for all x ϵ U

B. *Rough Intuitionistic Fuzzy Sets*

Extending the notion of rough fuzzy sets introduced by Dubois and Prade, rough intuitionistic fuzzy sets can be defined as follows.

Let (U, R) be an approximation space. Then for any X IF(U), the lower and upper approximations of X with respect to R are given by

(B.1) $M(\underline{R}X)(x) = \inf_{y \in [x]_R} MX(y)$ and $N(\underline{R}X)(x) = \sup_{y \in [x]_R} NX(y)$ for all xϵU and

(B.2) $M(\overline{R}X)(x) = \sup_{y \in [x]_R} MX(y)$ and $N(\overline{R}X)(x) = \inf_{y \in [x]_R} NX(y)$ for all xϵU.

C. *Rough Intuitionistic Fuzzy Sets Model Based on Multi-Granulations*

In this section we extend the concept of rough fuzzy sets on multigranulation of Wu and Kou [104] to introduce rough intuitionistic fuzzy sets on multigranulation as follows.

Definition C.1: Let K = (U, R) be a knowledge base, R be a family of equivalence relations on U and P, Q ϵ R. $(\underline{P+Q})((\overline{P+Q})(X))$. For ∀X ϵ IF(U) , the lower approximation $(\underline{P+Q})(X)$ and upper approximation $(\overline{P+Q})(X)$ of X based equivalence relations P, Q are defined as follows. For ∀x ϵ U,

(C.1) $M(\underline{P+Q})(X)(x) = \inf_{y \in [x]P} MX(y) \vee \inf_{y \in [x]Q} MX(y)$,

$N(\underline{P+Q})(X)(x) = \sup_{y \in [x]P} NX(y) \wedge \sup_{y \in [x]Q} NX(y)$

(C.2) $\forall x \in U, M(\overline{P+Q})(X)(x) = (M(\underline{P+Q})(X^C))^C(x)$ and $N(\overline{P+Q})(X)(x) = (N(\underline{P+Q})(X^C))^C(x)$.

(C.3) $(\underline{P+Q})(X)(x) = (M(\underline{P+Q})(X)(x), N(\underline{P+Q})(X)(x))$,

$(\overline{P+Q})(X)(x) = (M(\overline{P+Q})(X)(x), N(\overline{P+Q})(X)(x))$.

If $(\underline{P+Q})(X) = (\overline{P+Q})(X)$ then X is called definable, otherwise X is called an intuitionistic fuzzy rough set with respect to multi-granulations P and Q. The pair $((\underline{P+Q})(X), (\overline{P+Q})(X))$ is called a MG-intuitionistic fuzzy rough set on multi-granulations P and Q.

IV. Multi Granulation In Incomplete Information Systems

An information system is an ordered triplet S = (U, AT, f), where U is a finite nonempty set of objects, AT is a finite nonempty set of attributes and f: U → V_a, for any $a \in AT$, where V_a is the domain of any attribute a. In particular, a target information system (IS) is given by S = (U, AT, f, D, g), where D is a finite nonempty set of decision attributes and : $g_d: U \to V_d$ for any $d \in D$, where V_d is the domain of a decision attribute d.

For an IS, any attribute domain V_a may contain the special symbol "*" to indicate that the value of an attribute is unknown. Any domain value different from "*" is called regular.

Definition IV.1: A system in which values of all attributes for all objects from U are regular (known) is called complete and is called incomplete otherwise.

A. *MGRS in Incomplete Information system*

An information system is a pair S = (U, AT, f, D, g) is called an incomplete target IS if values of some attributes in AT are missing and those of all attributes in D are regular (known), where AT is called the set of conditional attributes and D is the set of decision attributes.

Definition A.1: Let S = (*U*, *A*) be an incomplete information system, P ⊆ A an attribute set. We define a binary relation on U as follows

(A.1) SIM(P) = {(u, v) ϵ U×U | ∀a ϵ P, a(u) = a(v) or a(u) = * or a(v) = *}.

If the attributes $P \subseteq AT$ are numerical attributes, we define SIM(P) relation as follows:

(A.2) SIM(P) = {(u, v)ϵU×U | ∀a ϵ P, $|a(u) - a(v)| \le \delta_a$ or a(u) = * or a(v) = *}.

In fact, SIM(P) is a tolerance relation on U. The concept of a tolerance relation has a wide variety of applications in classifications [46, 47, 48]. It can be shown that

(A.3) $SIM(P) = \bigcap_{a \in P} SIM(\{a\})$

Let $S_P(u)$ denote the set {v ϵ U| (u,v) ϵ SIM(P)}. $S_P(u)$ is the maximal set of objects which are possibly indistinguishable by P with u.

Let U/SIM(P) = { $S_P(u)$ | u ϵ U}, the classification or the knowledge induced by P. A member $S_P(u)$ from U/SIM(P) will be called a tolerance class or an information granule. It should be noticed that the tolerance classes in U/SIM(P) do not constitute a partition of U in general. They constitute a cover of U, i.e., $S_P(u) \neq \phi$ for every u ϵ U, and $\bigcup_{u \in U} S_P(u) = U$.

Next we define incomplete MGRS on Two Granulation Spaces.

Definition A.2: Let S = (U, AT, f) be an incomplete information system. Let P, Q ⊆ AT be two attribute subsets and X ⊆ U. We define the lower approximation of X and the upper approximation of X in U by the following:

(A.4) $\underline{P+Q}X = \bigcup \{x \mid S_P(x) \subseteq X \text{ or } S_Q(x) \subseteq X\}$and

(A.5) $\overline{P+Q}(X) = (\underline{P+Q}(X^c))^c$

The ordered pair $((\underline{P+Q})(X), (\overline{P+Q})(X))$ is called a rough set of X with respect to P+Q. The area of uncertainty or boundary region of this rough set is defined by

(A.6) $BN_{(P+Q)}(X) = (\overline{P+Q})(X) \setminus (\underline{P+Q})(X)$

Property A.1: Let S = (U, AT, f) be an incomplete IS, X ⊆ U and P, Q ⊆ AT be two-attribute subsets. Then the properties (2.4.3) to (2.4.11) hold true.

B. MGRFS in Incomplete Information systems

In this section we generalise both the MGRFS and MGRS on incomplete information systems to introduce the concept of MGRFS in incomplete information systems.

Let S = (U, AT, f) be an incomplete target IS and P, Q ⊆ AT two-attribute subsets, and X ϵ F(U). Then a lower approximation of X in U is defined by

(B.1) $(\underline{P+Q})(X)(x) = \inf_{y \in Sp(x)} X(y) \vee \inf_{y \in SQ(x)} X(y), \forall x \in U;$

(B.2) $(\overline{P+Q})(X)(x) = ((\underline{P+Q})(X^C))^C(x), \forall x \in U.$

Definition B.1: The ordered pair $((\underline{P+Q})(X), (\overline{P+Q})(X))$ is called a MG-fuzzy rough set of X based on multigranulations P and Q. The area of uncertainty or boundary region of this MG-fuzzy rough set is given by

(B.3) $BN_{P+Q}(X) = (\overline{P+Q})(X) \setminus (\underline{P+Q})(X).$

C. MGRIFS in an Incomplete Information system

In this section we generalise both the MGRFS and MGRS on incomplete information systems to introduce the concept of MGRIFS in incomplete information systems.

Let S = (U, AT, f) be an incomplete target IS and P, Q ⊆ AT two-attribute subsets, and X ϵ IF(U). Then a lower approximation of X in U is defined by

(C.1) $M(\underline{P+Q})(X)(x) = \inf_{y \in Sp(x)} MX(y) \vee \inf_{y \in SQ(x)} MX(y), \forall x \in U;$

(C.2) $N(\underline{P+Q})(X)(x) = \sup_{y \in Sp(x)} NX(y) \wedge \sup_{y \in SQ(x)} NX(y), \forall x \in U;$

(C.3) $M(\overline{P+Q})(X)(x) = (M(\underline{P+Q})(X^C))^C(x), \forall x \in U.$

(C.4) $N(\overline{P+Q})(X)(x) = (N(\underline{P+Q})(X^C))^C(x), \forall x \in U.$

(C.5) $(\underline{P+Q})(X)(x) = (M(\underline{P+Q})(X)(x), N(\underline{P+Q})(X)(x)),$
$(\overline{P+Q})(X)(x) = (M(\overline{P+Q})(X)(x), N(\overline{P+Q})(X)(x)), \forall x \in U.$

Definition C.1: The ordered pair $((\underline{P+Q})(X), (\overline{P+Q})(X))$ is called a MG-intuitionistic fuzzy rough set of X based on multi-granulations P and Q. The area of uncertainty or boundary region of this MG-intuitionistic fuzzy rough set is given by

(C.6) $BN_{P+Q}(X) = (\overline{P+Q})(X) \setminus (\underline{P+Q})(X).$

We have the following equivalent definition of $BN_{P+Q}(X)$:

$$BN_{P+Q} = (\overline{P+Q})(X) \bigcap ((\underline{P+Q})(X))^C$$
$$= (M(\overline{P+Q})(X), N(\overline{P+Q})(X)) \bigcap (M(\underline{P+Q})(X), N(\underline{P+Q})(X))^C$$
$$= (M(\overline{P+Q})(X), N(\overline{P+Q})(X)) \bigcap (N(\underline{P+Q})(X), M(\underline{P+Q})(X))^C$$
$$= (\min(M(\overline{P+Q})(X), N(\underline{P+Q})(X)), \max(N(\overline{P+Q})(X), M(\underline{P+Q})(X)))$$

Proposition C.1: Let S = (U, AT, f) be an incomplete target IS and P, Q ⊆ AT two-attribute subsets, and X ϵ F(U). Then the properties (2.4.3) to (2.4.11) hold true.

Using the above properties, we shall establish some topological properties of MGRIFSs. We explain below how the lower and upper approximations can be computed for MGRIFSs.

Example 1: Let us consider the following incomplete target IS about an emporium investment project.

Table I

Project	Locus	Investment	Population Density	Decision
x_1	common	high	0.88	yes
x_2	Bad	high	*	yes
x_3	Bad	*	0.33	no
x_4	Bad	low	0.40	no
x_5	Bad	low	0.37	no
x_6	Bad	*	0.60	yes
x_7	common	high	0.65	no
x_8	Good	*	0.62	yes

Let K = (U, AT, f, D), where U = $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\}$ and AT = {L, I, P}.
U/SIM (L) = $\{\{x_1,x_7\},\{x_2,x_3,x_4,x_5,x_6\},\{x_8\}$ and
U/SIM (P) = $\{\{x_1,x_2\},\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\},\{x_2,x_6,x_7,x_8\}\}$.
Suppose, X = $\{(x_1,0.5,0.3), (x_2,0.3,0.4), (x_3,0.3,0.5), (x_4,0.6,0.3), (x_5,0.5,0.4), (x_6,0.8,0.2), (x_7,1,0), (x_8,0.8,0.1)\}$.

$(\underline{L+P})(X) =$ $\{(x_1,0.5,0.3), (x_2,0.3,0.5), (x_3,0.3,0.5), (x_4,0.3,0.5), (x_5,0.3,0.5), (x_6,0.3,0.5), (x_7,0.5,0.3), (x_8,0.8,0.1)\}$

$X^C =$ $\{(x_1,0.3,0.5), (x_2,0.4,0.3), (x_3,0.5,0.3), (x_4,0.3,0.6), (x_5,0.4,0.5), (x_6,0.2,0.8), (x_7,0,1), (x_8,0.1,0.8)\}$

$(\overline{L+P})(X) =$ $\{(x_1,1,0), (x_2,0.8,0.2), (x_3,0.8,0.2), (x_4,0.8,0.2), (x_5,0.8,0.2), (x_6,0.8,0.2), (x_7,1,0), (x_8,0.8,0.1)\}$

V. Topological Properties of MGFRS in an Incomplete Information System

It has been noted by Pawlak that in the practical applications of rough sets two characteristics are very important. These are the accuracy measure and the topological characterization. The topological characterisation of rough sets depends upon the four types of rough sets. Following this approach, we define below four types of MGRIFS in an incomplete information system. Here, we denote by the strict one cut of an intuitionistic fuzzy set A by $A_{<1}$ and it contains all the elements of U which have non- membership value strictly less than one. It may be noted that in the case of a fuzzy set, this is equivalent to the support set of A, which comprises of elements having positive membership value.

Definition V.1: A MGFRS in an incomplete information space can be classified into following four types

(V.1) If $(\underline{P+Q}(X))_{<1} \neq \phi$ and $(\overline{P+Q}(X))_{<1} \neq U$, then we say that X is roughly P+Q-definable (Type-1/T-1).

(V.2) If $(\underline{P+Q}(X))_{<1} = \phi$ and $(\overline{P+Q}(X))_{<1} \neq U$, then we say that X is internally P+Q-undefinable (Type-2/T-2).

(V.3) If $(\underline{P+Q}(X))_{<1} \neq \phi$ and $(\overline{P+Q}(X))_{<1} = U$, then we say that X is externally P+Q–undefinable (Type-3/T-3).

(V.4) If $(\underline{P+Q}(X))_{<1} = \phi$ and $(\overline{P+Q}(X))_{<1} = U$, then we say that X is totally P+Q –undefinable (Type-4/T-4).

It is worth noting that when we consider X as a crisp set or a fuzzy set instead of an intuitionistic fuzzy set then the above definitions reduce to their counterpart in the crisp rough set or fuzzy rough set concept respectively.

A. Results

In this section we shall find out the types of MGRIFS in incomplete information systems. There are four sets of results accumulated in four tables. The first provides the type of a P+Q MGRIFS from the types of P and Q. The second table provides the types of the complement of a MGRIFS. In the third table we obtain the types for the union of two MGRIFSs of all possible types. Similarly we establish the types of the intersection of two MGRIFSs of all possible types. These results will be useful for further studies in approximation of classifications and rule generation.

1) Table for type of X with respect to P+Q

Table II

	Type of X with respect to P				
Type of X with respect to Q		T-1	T-2	T-3	T-4
	T-1	T-1	T-1	T-1	T-1
	T-2	T-1	T-2	T-1	T-2
	T-3	T-1	T-1	T-3	T-3
	T-4	T-1	T-2	T-3	T-4

2) Table for type of X^C with respect to P+Q

Table III

X	X^C
T-1	T-1
T-2	T-3
T-3	T-2
T-4	T-4

3) Table for type of X ⋃ Y with respect to P+Q

Table IV

	Type of Y with respect to P+Q				
Type of X with respect to P+Q		T-1	T-2	T-3	T-4
	T-1	T-1/T-3	T-1/T-3	T-3	T-3
	T-2	T-1/T-3	T-1/T-2/T-3/T-4	T-3	T-3/T-4
	T-3	T-3	T-3	T-3	T-3
	T-4	T-3	T-3/T-4	T-3	T-3/T-4

We shall provide an example to show that for two multigranular rough intuitionistic fuzzy sets of Type 1, the union can be of Type 1 or Type 3. The other cases can be justified in a similar manner.

Example A.1 Let us consider the example 5.1 above.

Suppose,
X = $\{(x_1,0,1),(x_2,0.3, 0.6), (x_3,0,0.6), (x_4,0,1), (x_5,0,0.6), (x_6,1,0), (x=,0,1),(x_8,1,0)\}$ and
Y = $\{(x_1,0,1), (x_2,0.3,0.5), (x_3,0,1), (x_4,0,1), (x_5,0,0.9), (x_6,0,0.9), (x_7,0,1),(x_8,0,0.8)\}$.
Then X and Y are both of Type 1 as
$(\underline{L+P})(X)=\{(x_1,0,1), (x_2,0,1), (x_3,0,1), (x_4,0,1), (x_5,0,1), (x_6,0,1), (x_7,0,1),(x_8,1,0)\}$. So that $((\underline{L+P})(X))_{<1} \neq \phi$
$(\underline{L+P})(Y) = \{(x_1,0,1), (x_2,0,1),(x_3,0,1),(x_4,0,1),(x_5,0,1), (x_6,0,1), (x_7,0,1), (x_8,0,0.8)\}$. So that $((\underline{L+P})(Y))_{<1} \neq \phi$,

$(\overline{L+P})(X)$ = {(x_1,0,1), (x_2,1,0),(x_3,1,0),(x_4,1,0),(x_5,1,0), (x_6,1,0), (x_7,0,1), (x_8 ,1,0)}. So that $((\overline{L+P})(X))_{<1} \neq U$ and

$(\overline{L+P})(Y)$ = {(x_1,0,1), (x_2,0.3,0.5), (x_3,0.3,0.5), (x_4,0.3,0.5), (x_5,0.3,0.5), (x_6 ,0.3,0.5),(x_7 ,0,1),(x_8,0,0.8)}.

So that $((\overline{L+P})(Y))_{<1} \neq U$. Now we have
$X \cup Y$= {(x_1,0,1), (x_2,0.3,0.5),(x_3,0,0.6),(x_4,0,1),(x_5,0,0.6), (x_6,1,0), (x_7,0,1),(x_8 ,1, 0)}.
So that$(\underline{L+P})X \cup Y$ = {(x_1,0,1),(x_2,0,1),(x_3,0,1), (x_4,0,1), (x_5,0,1), (x_6,0,1),(x_7,0,1), (x_8,1,0)}.
So that $((\underline{L+P})(X \cup Y))_{<1} \neq U$ and

$(\overline{L+P})(X \cup Y)$ = {(x_1,0,1), (x_2,1,0),(x_3,1,0),(x_4,1,0), (x_5,1,0), (x_6,1,0), (x_7,0,1),(x_8 ,0,1)}.

So that $((\overline{L+P})(X \cup Y))_{<1} \neq U$
So, XUY is of Type 1.
Next, we take X = {(x_1,0,1), (x_2,0,0.6),(x_3,0,0.6), (x_4,0,0.6), (x_5,0,0.6),(x_6 ,0.8,0.1),(x_7,0,0.9),(x_8,0,1)} and Y = {(x_1,0,1), (x_2,0,0.6),(x_3,0,0.6),(x_4,0,0.6), (x_5,0,0.6), (x_6,0,0.6),(x_7,0,1),(x_8 ,0.8,0.1)} . Both X and Y are of Type 1 as $(\underline{L+P})(X)$ ={(x_1,0,1), (x_2,0,0.6),(x_3,0,0.6),(x_4,0,0.6), (x_5,0,0.6), (x_6,0,0.6),(x_7,0,1),(x_8,0,1)}.
So that $((\underline{L+P})(X))_{<1} \neq \phi$

$(\overline{L+P})(X)$ = {(x_1,0,0.9), (x_2,0.8,0.1),(x_3,0.8,0.1),(x_4,0.8,0.1), (x_5,0.8,0.1),(x_6 ,0.8,0.1),(x_7,0,0.9),(x_8,0,1)}. ,

So that $((\overline{L+P})(X))_{<1} \neq U$ and $(\underline{L+P})(Y)$ = {(x_1,0,1),(x_2,0,0.6), (x_3,0,0.6), (x_4,0,0.6),(x_5,0,0.6),(x_6,0,0.6),(x_7,0,1),(x_8,0.8,0.1)}. So that $((\underline{L+P})(Y))_{<1} \neq \phi$

$(\overline{L+P})(Y)$ = {(x_1,0,1), (x_2,0,0.6),(x_3,0,0.6), (x_4,0,0.6), (x_5,0,0.6), (x_6,0,0.6),(x_7 ,0,1),(x_8,0,0.8,0.1)}.

So that $((\overline{L+P})(Y))_{<1} \neq U.$
Now, $(\underline{L+P})(X \cup Y)$ = {(x_1,0,1), (x_2,0,0.1),(x_3,0,0.1),(x_4,0,0.1), (x_5,0,0.1),(x_6 ,0,0.1),(x_7 ,0,1),(x_8 ,0.8,0.1)}.
So that $((\underline{L+P})(X \cup Y))_{<1} \neq \phi$

$(\overline{L+P})(X \cup Y)$ = {(x_1,0,0.9), (x_2,0.8,0.1),(x_3,0.8,0.1), (x_4,0.8,0.1), (x_5,0.8,0.1),(x_6 ,0.8,0.1),(x_7,0,0.9),(x_8,0.8,0.1)}. So that $((\overline{L+P})(X \cup Y))_{<1} = U$ .So XUY is of Type 3.
Hence both the cases in the table position (1, 1) are possibilities.

Proof of entry (1, 3)
Let X and Y be of Type 1 and Type 3 respectively. Then from the properties of Type 1 and Type 3 of MGFRS, we have

$((\underline{P+Q})(X))_{<1} \neq \phi$, $((\underline{P+Q})(Y))_{<1} \neq \phi$, $((\overline{P+Q})(X))_{<1} \neq U$ and $((\overline{P+Q})(Y))_{<1} = U$. So, using (2.4.13) and (2.4.15) we get $((\underline{P+Q})(X \cup Y))_{<1} \neq \phi$ and $((\overline{P+Q})(X \cup Y))_{<1} = U$. Hence X ∪Y is of type 3 only.

4) Table for type of $X \cap Y$ with respect to P+Q

Table V

		Type of Y with respect to P+Q			
Type of X with respect to P+Q		T-1	T-2	T-3	T-4
	T-1	T-1/T-2	T-2	T-1/T-2	T-2
	T-2	T-2	T-2	T-2	T-2
	T-3	T-1/T-2	T-2	T-1/T-2/T-3/T-4	T-2/T-4
	T-4	T-2	T-2	T-2/T-4	T-2/T-4

We shall provide an example to show that for two multigranular rough sets such that one is of Type 1 and the other one is of Type 3.The intersection can be of Type 1 or Type 2. The other cases can be justified in a similar manner.

Example A.2
Let us consider the example 4.1 above. We take
X = {(x_1,0,1), (x_2,0.3,0.6), (x_3,0,0.9), (x_4,0,0.9), (x_5,0,0.9), (x_6,0.8,0.1), (x_7,0,1), (x_8,0.8,0.1)} and
Y = {(x_1,1,0), (x_2,0,0.9), (x_3,0,0.9), (x_4,0,0.9), (x_5,0,0.9), (x_6,0,0.9), (x_7,0,0.9), (x_8,0.8,0.1)}.
Then X and Y are of Type 3 and Type 1 respectively as $(\underline{L+P})(X)$ = {(x_1,0,1), (x_2,0,0.9), (x_3,0,0.9), (x_4,0,0.9), (x_5,0,0.9), (x_6,0,0.9), (x_7,0,1),(x_8,0.8,0.1)}.
So that $((\underline{L+P})(X))_{<1} \neq \phi$ and $(\underline{L+P})(Y)$ = {(x_1,0,0.9), (x_2,0,0.9),(x_3,0,0.9),(x_4,0,0.9),(x_5,0,0.9),(x_6,0,0.9),(x_7,1,0), (x_8 ,0.8,0.1).

So that $((\underline{L+P})(Y))_{<1} \neq \phi$. Also, $(\overline{L+P})(X)$ = {(x_1,0.8,0.1), (x_2,0.8,0.1),(x_3,0.8,0.1),(x_4,0.8,0.1),(x_5,0.8,0.1),(x_6,0.8,0.1), (x_7,0.8,0.1),(x_8,0.8,0.1)}.

So that $((\overline{L+P})(X))_{<1} = U$ and $(\overline{L+P})(Y)$ = {(x_1,1,0), (x_2,0,1),(x_3,0,1),(x_4,0,1),(x_5,0,1),(x_6,0,1),(x_7,1,0),(x_8 ,0.8,0. 1)}. So that $(\overline{L+P})(X))_{<1} \neq U.$
We have X∩Y = {(x_1,0), (x_2,0), (x_3,0), (x_4,0), (x_5,0), (x_6,0), (x_7,0), (x_8,0.8)}.
Hence $(\underline{L+P})(X \cap Y)$ = {(x_1,0,1), (x_2,0,1), (x_3,0,1), (x_4,0,1), (x_5,0,1), (x_6,0,1),(x_7,0,1),(x_8,0.8,0.1)}. 1 So that $((\underline{L+P})(X \cap Y))_{<1} \neq \phi$.

$(\overline{L+P})(X \cap Y)$ = {(x_1,0,1), (x_2,0,0.9),(x_3,0,0.9),(x_4,0,0.9),(x_5,0,0.9),(x_6,1,0), (x_7,0,1), (x_8,0.8,0.1)}.

So that $((\overline{L+P})(X \cap Y))_{<1} \neq U.$
Hence $X \cap Y$ is of Type 1.
Again taking X = {(x_1,0,1), (x_2,0,1),(x_3,0.3,0.6), (x_4,0.6,0.3), (x_5,0,1), (x_6,0,0.9), (x_7,0,0.9),(x_8,0.8,0.1)}and Y = {(x_1,0.3,0.6), (x_2,0,1), (x_3,0,1), (x_4,0.6,0.2), (x_5,0.5,0.3), (x_6,0,1), (x_7,0.4,0.5), (x_8,0.8,0.1)},
we find that X and Y are of Type3 and Type1 respectively as detailed below.
$(\underline{L+P})(X)$ = {(x_1,0,1), (x_2,0,0.9), (x_3,0,0.9), (x_4,0,0.9), (x_5,0,0.9), (x_6,0,0.9), (x_7,0,1),(x_8,0.8,0.1)} .
So that $((\underline{L+P})(X))_{<1} \neq \phi$
$(\underline{L+P})(Y)$ = {(x_1,0.3,0.6), (x_2,0,1),(x_3,0,1),(x_4,0,1),(x_5,0,1), (x_6,0,1), (x_7,0.3,0.6), (x_8 ,0.8,0.1)}.
So that $((\underline{L+P})(Y))_{<1} \neq \phi$

$(\overline{L+P})(X)$ = {(x_1,0,1), (x_2,0.6,0.3),(x_3,0.4,0.5),(x_4,0.6,0.3), (x_5,0.6,0.3), (x_6,0.6,0.3),(x_7,0,1),(x_8,0.8,0.1)}.

So that $((\overline{L+P})(X))_{<1} \neq U$. and

$(\overline{L+P})(Y)$ = {(x_1,0.4,0.5), (x_2,0.6,0.3),(x_3,0.6,0.3), (x_4,0.6,0.3), (x_5,0.6,0.3), (x_6,0.6,0.3),(x_7,0.4,0.5),(x_8,0.2,0.7)}.

So that $((\overline{L+P})(X))_{<1} = U$. Now, $(\underline{L+P})(X\cap Y)$ = {(x_1,0,0.9), (x_2,0,0.9),(x_3,0,0.9), (x_4,0,0.9), (x_5,0,0.8), (x_6,0,0.8), (x_7,0,0.8),(x_8,0,0.8)}.

So that $(\underline{L+P})(X \cap Y)_{<1} = \phi$ and $(\overline{L+P})(X \cap Y)$ ={(x_1,0,1), (x_2,0.6,0.3),(x_3,0.6,0.3),(x_4,0.6,0.3),(x_5,0.6,0.3),(x_6,0.6,0.3), (x_7,0,1),(x_8 ,0,1)}.

So that $((\overline{L+P})(X))_{<1} \neq U$.

So, $X\cap Y$ is of Type 2. Hence both the cases for intersection operation in position (3, 1) can occur.

Proof of entry (2, 1)

Let X and Y be of Type 2 and Type 1 respectively. Then from the properties of Type2 and Type1 MGFRSs we get $((\underline{P+Q})(X))_{<1} = \phi$, $((\underline{P+Q})(Y))_{<1} = \phi$, $((\overline{P+Q})(X))_{<1} \neq U$ and $((\overline{P+Q})(Y))_{<1} \neq U$. So using properties (2.4.11) and (2.4.14) we get $((\underline{P+Q})(X\cap Y))_{<1} = \phi$ and $((\overline{P+Q})(X\cap Y))_{<1} \neq U$. So, $X\cap Y$ is of Type 2. This completes the proof. The other cases can be established similarly.

VI. Conclusions

In this paper we introduced the concept of multigranular intuitionistic fuzzy rough sets in incomplete information systems and established many properties of these sets. Also, we studied the topological properties of MGIFRSs with respect to the three set theoretic operations of union, intersection and complementation. The tables show that there are multiple answers to some of the cases as like as the case of basic rough sets or fuzzy rough sets. Also, we provided examples in some cases to illustrate the fact that the multiple answers can actually occur. These results can be used in approximation of classifications and rule induction.

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