Analytic Hierarchy Process & TOPSIS Method to Evaluate Faculty Performance in Engineering Education

Dipendra Nath Ghosh

Department of Computer Science & Engineering
Dr. B.C Roy Engineering College, Jemua Road, Fuljhore, Durgapur-713206
West Bengal, India

***Abstract*-Evaluation of faculty performance is a multi-criteria decision making (MCDM) problem. Best faculty members are the assets of an educational organization for increasing the quality education as well as for the popularity of that organization. So the performance evaluation of teachers is very much essential in education to enrich their knowledge and to assess each individual's contribution to the organization. In this paper a study has been made by applying Analytic Hierarchy Process (AHP) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) to evaluate faculty performance in engineering education. Here first ten students feedback opinion of a particular department have been considered to evaluate four teachers performances based on the criteria: subject knowledge, method of teaching, communication skill, accessibility, discipline & behavior, power of explanation and attitude. The proposed model yields the ranking of the four faculty members for evaluating their performances.**

***Keywords*-Multi-Criteria Decision Analysis, Analytical Hierarchy Process, TOPSIS model.**

I. INTRODUCTION

Education system specially engineering education contributes a major role to develop the nation. In an academic institution Teachers and Students are two main pillars and without these two an academic organization can never be survived. Teachers are the most important assets of an educational organization and good teachers provide the good quality education among the students. It means that teachers' performance evaluation has become one of the most important activities not only for the long run of an organization but for the development of the society.

Evaluation of best technical institutions fuzzy analytical hierarchy process was developed to tolerate vagueness and uncertainty of human judgment [8]. To improve the lack of recruitment processes as well as reduce individual senses of supervisory level by fuzzy logic and AHP methods, Pin-Chang Chen tried [9] to identify appropriate personality traits and key professional skills through the information statistics. P. Kousalya and et.al. presented the use of multi criteria decision-making methods for ranking alternatives that curb student absenteeism in engineering college [10]. S. Mahmoodzadeh and et.al proposed a new methodology to provide a simple approach to assess alternative projects and help decision makers to select the best one with the help of fuzzy AHP and TOPSIS technique [11].

In this paper, performance evaluation of faculty members for an educational organization is presented. The proposed approach is based on two-step AHP and TOPSIS methodology for selecting the best teaching associate which is more powerful than traditional methods. Several students gave their opinions for teachers depending upon different criteria.

II. CONCEPT OF AHP & TOPSIS

A. Analytic Hierarchy Process (AHP)

The pair-wise comparison method and the hierarchical model were developed in 1980 by T.L.Saaty in the context of the Analytical Hierarchy Process (AHP) [1, 2]. It is one of the best and most widely used MCDM approaches. AHP is an approach to decision making that involves structuring multiple choice criteria into a hierarchy, assessing the relative importance of these criteria, comparing alternatives for each criterion and determining an overall ranking of the alternatives [3]. AHP helps to capture both subjective and objective evaluation measures, providing a useful mechanism for checking the consistency of the evaluation measures and alternatives suggested by the team thus reducing bias in decision making [5]. Some of its applications include technology choice [7], vendor selection of a telecommunications system [6], project selection, budget allocation. The steps for implementing the AHP process are illustrated [4] as follows:

- Define the Objectives.

- Identify the Criteria/Attributes.
- Choose the Alternatives.
- Establish the Hierarchy.
- Design Questionnaire and survey
- Construct the Pairwise Comparison matrices using Satty's 9-point scale.

TABLE. 1: SAATY'S 9-POINT SCALE OF PAIR-WISE COMPARISON

Scale	**Compare factor of i and j**
1	Equally Important
3	Weakly Important
5	Strongly Important
7	Very Strongly Important
9	Extremely Important
2,4,6,8	Intermediate value between adjacent scales

- Synthesize Judgments.
- Calculate Consistency (C.I) Index.

TABLE. 2: TABLE OF RANDOM INDEX (SAATY, 1980)

MATRIX ORDER	1,2	3	4	5	6	7
R.I.	0	0.52	0.89	1.12	1.26	1.36
MATRIX ORDER	8	9	10	11	12	13
R.I.	1.41	1.46	1.49	1.52	1.54	1.56

- Comparison between Criteria and Alternatives
- Calculate Final Rankings

An AHP hierarchy has at least three levels:
Level-1: The main objective or goal of the problem at the top.
Level-2: Multiple criteria that define alternatives in the middle.
Level-3: Competing alternatives at the bottom.

In the proposed model, AHP hierarchy for best teacher selection through student's feedback report is shown in Fig. 1.

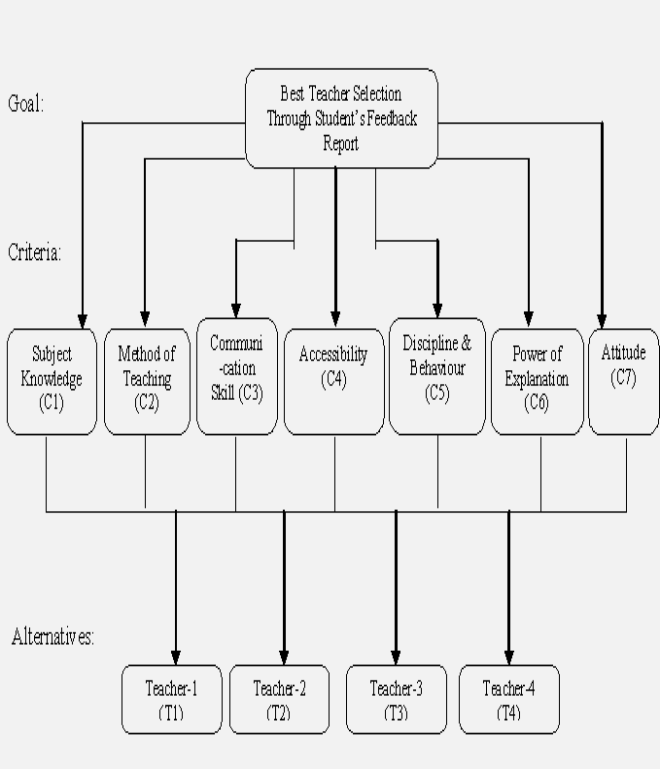

Fig. 1: Hierarchy of Best Teacher Selection

B. TECHNIQUE FOR ORDER PREFERENCE BY SIMILARITY TO IDEAL SOLUTION (TOPSIS)

TOPSIS, known as one of the most classical MCDM methods, was first developed by Hwang and Yoon [13], is based on the idea that the chosen alternative should have the shortest distance from the Positive Ideal Solution (PIS) and on the other side the farthest distance of the Negative Ideal Solution (NIS). The Positive Ideal Solution maximizes the benefit criteria and minimizes the cost criteria, whereas the Negative Ideal Solution maximizes the cost criteria and minimizes the benefit criteria [16, 17]. In the process of TOPSIS, the performance ratings and the weights of the criteria are given as exact values. Abo-sinna and Amer [12] extend TOPSIS approach to solve multi-objective nonlinear programming problems. Jahanshahloo et al. [14] extends the concept of TOPSIS to develop a methodology for solving multi-criteria decision-making problems with interval data. The steps of TOPSIS model are as follows:

- Calculate the normalized decision matrix.
- Calculate the weighted normalized decision matrix.
- Determine the Positive Ideal Solution and Negative Ideal Solution.

- Calculate the separation measures for each alternative from the positive and negative ideal solution.
- Calculate the relative closeness to the ideal solution for each alternative.
- Rank the preference order.

III. TWO STEPS AHP & TOPSIS METHOD

In this paper we use two step methods consist of AHP and TOPSIS. In first step AHP is used for calculating the weights of the attributes or criteria as well as the overall weights of the candidates in each attribute. In second step these weights are considered and used in TOPSIS process. Then TOPSIS is applied for the evaluation problem and the result shows the preference order of the teachers in an educational organization. This methodology levels can be discussed clearly.

AHP:

Step1: Selection of Experts
Step2: Identify the Attributes/Criteria
Step3: Identify the Alternatives
Step4: Design the Hierarchy
Step5: Establish the pair-wise comparison of the Criteria
Step6: Calculate the Eigen value and Eigen vector
Step7: Perform the Consistency Test
Step8: Compute the weights of the Criteria
Step9: Establish the pair-wise comparison of the Alternatives with respect to each Criteria.
Step10: Calculate the Eigen value and Eigen vector for each of them.
Step11: Perform the Consistency Test
Step12: Compute the weights of the Alternatives for each Criteria
Step13: Calculate the Geometric Mean of the weights calculated by Experts
Step14: Calculate the Eigen value and Eigen vector
Step15: Perform the Consistency Test
Step16: Compute the overall weights of the Alternatives.

TOPSIS:

Step17: Start TOPSIS procedure using the weights calculated using AHP.
Step18: Calculate negative and positive ideal solutions & separation measures.
Step19: Rank the preference candidate in descending order.

IV. EXPERIMENTS & RESULTS

Students give their own remarks for each teacher for each criterion which is shown in the Table 3.

TABLE 3: 10- STUDENTS OPINION AGAINST EACH TEACHER

Criteria	**Subject Knowledge (C1)**									
Teacher	**S0**	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**S9**
T1	VG	VG	NG	VG	E	EG	E	EG	EG	E
T2	EG	EG	NG	EG	E	EG	VG	E	E	E
T3	G	G	G	VG	VG	G	VG	E	VG	E
T4	VG	G	NG	VG	E	EG	E	EG	EG	E
Criteria	**Method of Teaching (C2)**									
Teacher	**S0**	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**S9**
T1	VG	G	G	G	G	VG	E	VG	VG	E
T2	NG	B	NG	B	B	VG	E	VG	NG	EG
T3	E	EG	E	EG	EG	G	E	EG	E	E
T4	G	G	EG	G	G	VB	G	VG	G	EG
Criteria	**Communication Skill (C3)**									
Teacher	**S0**	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**S9**
T1	VG	E	EG	EG	EG	G	G	VG	G	VG
T2	EG	E	E	E	EG	EG	VG	G	VG	EG
T3	G	EG	VG	EG	VG	G	B	G	NG	G
T4	VG	E	VG	EG	EG	G	G	VG	G	VG
Criteria	**Accessibility (C4)**									
Teacher	**S0**	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**S9**
T1	VG	G	G	G	G	G	VG	G	G	VG
T2	E	G	E	EG	EG	G	G	VG	E	VG
T3	VG	G	EG	G	EG	G	VG	G	G	VG
T4	G	NG	EG	B	NG	NG	NG	G	NG	G
Criteria	**Discipline & Behavior (C5)**									
Teacher	**S0**	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**S9**
T1	VG	VG	E	G	EG	VG	G	G	G	E
T2	EG	EG	E	EG	E	VG	VG	G	VG	EG
T3	VG	VG	E	EG	EG	G	G	VG	G	E
T4	VG	NG	E	G	E	VG	E	G	G	E
Criteria	**Power of Explanation (C6)**									
Teacher	**S0**	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**S9**
T1	VG	EG	G	E	VG	E	E	E	E	EG
T2	NG	G	B	VG	NG	VG	VG	VG	VG	G
T3	G	G	NG	EG	G	EG	G	EG	EG	VG
T4	VG	EG	G	EG	G	EG	E	EG	E	EG
Criteria	**Attitude (C7)**									
Teacher	**S0**	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**S9**
T1	VG	VG	G	EG	NG	G	G	VG	G	EG
T2	G	NG	B	VG	B	VG	G	G	NG	VG
T3	EG	EG	VG	E	NG	VG	VG	EG	VG	E
T4	EG	EG	VG	EG	G	VG	VG	VG	VG	EG

According our two step methodology the model is followed the following steps:

Start of AHP method

Step-1: From the Table 3. taking the pair wise comparison matrix A according to Saaty's scale mentioned in table-1 of Student-1 for the criteria Subject Knowledge (C1) is as follows:-

TABLE 4: PAIR WISE COMPARISON MATRIX FOR C1

Criteria-1	**T1**	**T2**	**T3**	**T4**
T1	1.00000	0.50000	2.00000	1.00000
T2	2.00000	1.00000	4.00000	3.00000
T3	0.50000	0.25000	1.00000	0.50000
T4	1.00000	0.33333	2.00000	1.00000

Step-2: Calculate the column sum $\sum_i C_{ij}$ for each column in the table 5.

TABLE 5: COLUMN SUM FOR C1

Criteria-1	T1	T2	T3	T4
T1	1.00000	0.50000	2.00000	1.00000
T2	2.00000	1.00000	4.00000	3.00000
T3	0.50000	0.25000	1.00000	0.50000
T4	1.00000	0.33333	2.00000	1.00000
Sum	4.5000	2.0833	9.0000	5.5000

Step-3: Standardized each cell by $X_{ij} = \frac{C_{ij}}{\sum_i C_{ij}}$

TABLE 6: STANDARDIZED MATRIX FOR C1

Criteria-1	T1	T2	T3	T4
T1	0.2222	0.2400	0.2222	0.1818
T2	0.4444	0.4800	0.4444	0.5455
T3	0.1111	0.1200	0.1111	0.0909
T4	0.2222	0.1600	0.2222	0.1818

Step-4: Calculate row sum by $R_i = \sum_j X_{ij}$ and weight by $W_i = \frac{R_i}{n}$; n = no. of candidates.

TABLE 7: CALCULATE THE ROW SUM & WEIGHT

Criteria-1	T1	T2	T3	T4	Sum	W
T1	0.2222	0.2400	0.2222	0.1818	0.8663	0.2166
T2	0.4444	0.4800	0.4444	0.5455	1.9143	0.4786
T3	0.1111	0.1200	0.1111	0.0909	0.4331	0.1083
T4	0.2222	0.1600	0.2222	0.1818	0.7863	0.1966

Step-5: Calculate the priority vector by $V_i = A.W_i$ for i = 1,2,..,n.

TABLE 8: PRIORITY VECTOR CALCULATION

Criteria-1	T1	T2	T3	T4	W	P.Vector
T1	1.00000	0.50000	2.00000	1.00000	0.2166	0.8690
T2	2.00000	1.00000	4.00000	3.00000	0.4786	1.9345
T3	0.50000	0.25000	1.00000	0.50000	0.1083	0.4345
T4	1.00000	0.33333	2.00000	1.00000	0.1966	0.7892
Sum	4.5000	2.0833	9.0000	5.5000	1.0000	4.0273

Step-6: Calculate $\lambda_i = \frac{V_i}{W_i}$ and calculate λ_{max} by averaging the λ_i's in the table 9 given below.

TABLE 9: CALCULATION OF λ_{MAX}

Criteria-1	T1	T2	T3	T4	W	P.Vector	Lamda	max
T1	1.00000	0.50000	2.00000	1.00000	0.2166	0.8690	4.0126	4.0206
T2	2.00000	1.00000	4.00000	3.00000	0.4786	1.9345	4.0422	
T3	0.50000	0.25000	1.00000	0.50000	0.1083	0.4345	4.0126	
T4	1.00000	0.33333	2.00000	1.00000	0.1966	0.7892	4.0151	
Sum	4.5000	2.0833	9.0000	5.5000	1.0000	4.0273		

Step-7: Calculate Consistency Index (C.I) and Consistency Ratio (C.R).

Here C.I.= $\frac{\lambda_{max} - n}{n-1}$ = 0.006873 and C.R. = $\frac{C.I}{R.I}$ = 0.007722

This value of C.R is less than the allowable value of 0.10. Therefore, the consistency of the judgment matrix is found to be within an acceptable tolerance. But if the consistency ratio is greater than 0.10 we need to revise the subjective judgment.

Step-8: Repeat Step-1 to Step- 7 for Student-2 to Student-10 and check the consistency ratio for every judgment matrix.

Step-9: Calculate the Geometric mean of each cell of Student-1 to Student-10 and repeat Step -1 to Step -7 for calculating the overall weight for each teacher for the criteria Subject Knowledge (C1).

The weight for each teacher against the criteria C1 is shown in Fig. 2.

TABLE 10: OVERALL WEIGHT OF THE TEACHERS

Overall	W
T1	0.256569
T2	0.343071
T3	0.159836
T4	0.240524

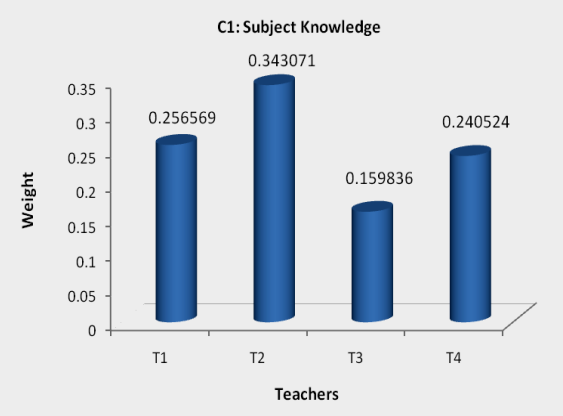

Fig. 2: Weight of teachers for criteria C1

Step -10**:** Repeat Step-1 to Step-9 for each criterion and calculate the weights for each teacher according to each criterion and calculate the weight of each criterion by step- 1 to step- 7

TABLE 11: OVERALL WEIGHT OF THE TEACHERS

Weight	0.30952	0.20661	0.10786	0.06532	0.10106	0.16433	0.04529
Teacher	C1	C2	C3	C4	C5	C6	C7
T1	0.256569	0.246876	0.24882	0.227471	0.220544	0.385853	0.19872
T2	0.343071	0.121642	0.38312	0.388753	0.318419	0.121412	0.115023
T3	0.159836	0.477826	0.134167	0.255007	0.252768	0.188865	0.356665
T4	0.240524	0.153656	0.233893	0.128769	0.208269	0.303871	0.329592

The weights for each teacher against the other criteria C2, C3, C4, C5, C6 and C7 are shown in figures below:

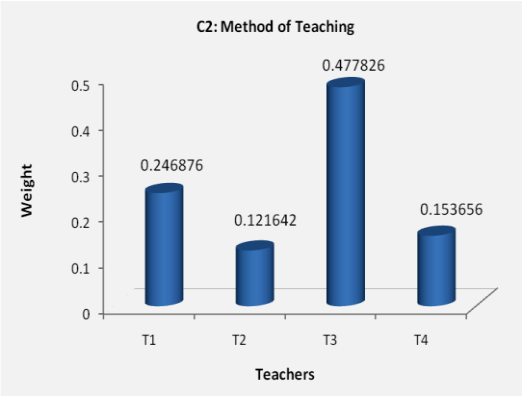

Fig. 3: Weight of teachers for criteria C2

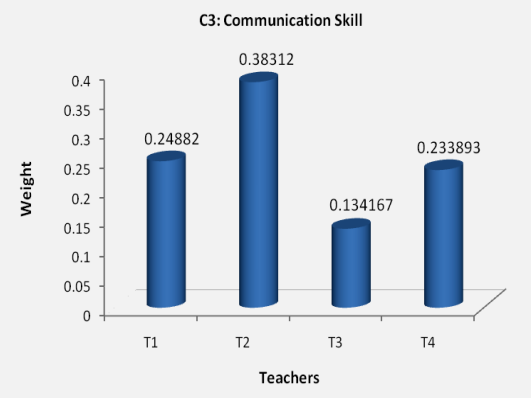

Fig. 4: Weight of teachers for criteria C3

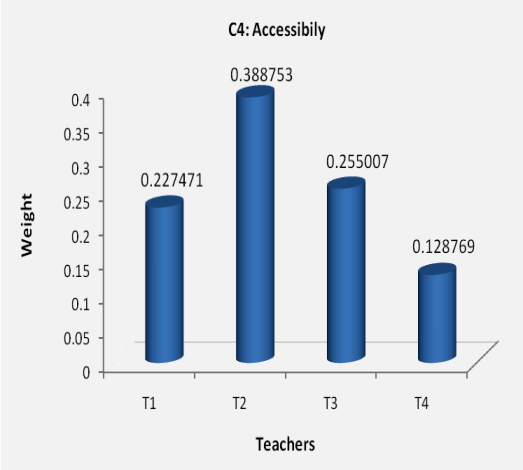

Fig. 5: Weight of teachers for criteria C4

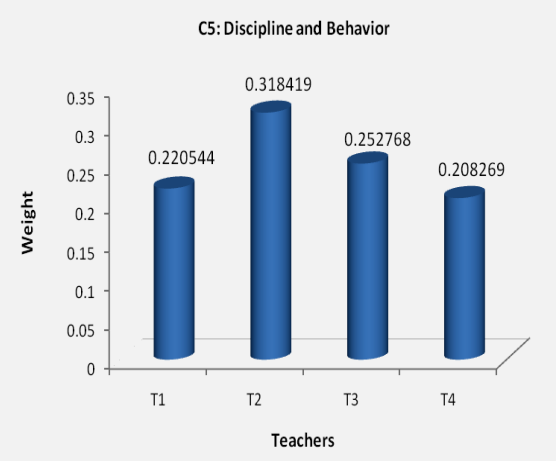

Fig. 6: Weight of teachers for criteria C5

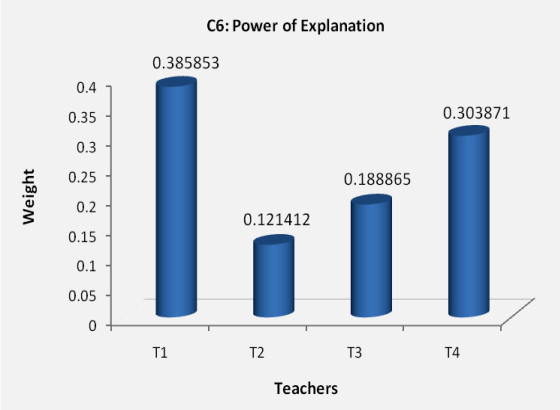

Fig. 7: Weight of teachers for criteria C6

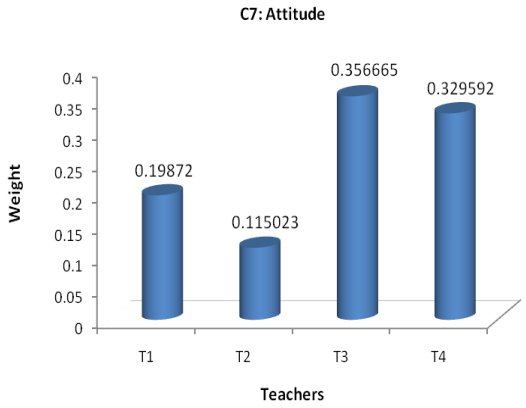

Fig. 8: Weight of teachers for criteria C7

Start of TOPSIS method

Step -11: Construct normalized decision matrix.
Normalize scores or data as follows:

$$r_{ij} = \frac{x_{ij}}{(\sum_i x_{ij}^2)^{\frac{1}{2}}} \text{ for i = 1, …, m; j = 1, …, n}$$

TABLE 12: NORMALIZED DECISION MATRIX

Weight	0.30952	0.20661	0.10786	0.06532	0.10106	0.16433	0.04529
Teacher	C1	C2	C3	C4	C5	C6	C7
T1	0.065828	0.060948	0.061911	0.051743	0.04864	0.148883	0.03949
T2	0.117698	0.014797	0.146781	0.151129	0.101391	0.014741	0.01323
T3	0.025548	0.228318	0.018001	0.065029	0.063892	0.03567	0.12721
T4	0.057852	0.02361	0.054706	0.016581	0.043376	0.092338	0.108631
SUM	0.26692	0.32767	0.2814	0.28448	0.2573	0.29163	0.28856
SQ. ROOT	0.51665	0.57243	0.53047	0.53337	0.50725	0.54003	0.53718

Weight	0.30952	0.20661	0.10786	0.06532	0.10106	0.16433	0.04529
Teacher	C1	C2	C3	C4	C5	C6	C7
T1	0.496604	0.43128	0.469055	0.42648	0.434788	0.714505	0.369933
T2	0.664033	0.212502	0.722227	0.728864	0.627741	0.224825	0.214124
T3	0.309371	0.834737	0.252921	0.478107	0.498315	0.349731	0.66396
T4	0.465548	0.268429	0.440916	0.241426	0.410588	0.562694	0.613561

Step -12: Construct the weighted normalized decision matrix. Multiply each column of the normalized decision matrix by its associated weight. An element of the new matrix is: $v_{ij} = w_j . r_{ij}$

TABLE 13: WEIGHTED NORMALIZED DECISION MATRIX

Weight	0.30952	0.20661	0.10786	0.06532	0.10106	0.16433	0.04529
Teacher	C1	C2	C3	C4	C5	C6	C7
T1	0.153709	0.089107	0.050592	0.027858	0.04394	0.117415	0.016754
T2	0.205531	0.043905	0.077899	0.047609	0.06344	0.036946	0.009698
T3	0.095757	0.172465	0.02728	0.03123	0.05036	0.057471	0.030071
T4	0.144096	0.05546	0.047557	0.01577	0.041494	0.092468	0.027788

Step -13: Determine the ideal and negative ideal solutions.
Ideal solution:-
A* = {v_1^*, …, v_n^*}, where
v_j^* ={ $\max_i(v_{ij})$ if j ∈ J ; $\min_i(v_{ij})$ if j∈J' }

= {0.205531, 0.172465, 0.077899, .047609, 0.06344, 0.117415, 0.030071}

Negative ideal solution:-
A' = {v_1', …, v_n' }, where
v_j' = { $\min_i(v_{ij})$ if j ∈ J ; $\max_i(v_{ij})$ if j ∈ J' }

= {0.095757, 0.043905, 0.02728, 0.01577, 0.041494, 0.036946, 0.009698}

Step -14: Calculate the separation measures for each alternative**.** The separation from the ideal alternative is:

$$S_i^* = [\sum_j (v_j^* - v_{ij})^2]^{\frac{1}{2}} \quad ; \ i = 1, \ldots, m$$

TABLE 14: SEPARATION MEASURE FROM IDEAL ALTERNATIVE

Teacher	C1	C2	C3	C4	C5	C6	C7	SUM	S_i^*
T1	0.002686	0.006949	0.000746	0.00039	0.00038	0	0.000178	0.011329	0.106438
T2	0	0.016528	0	0	0	0.006475	0.000415	0.023418	0.153029
T3	0.01205	0	0.002562	0.000268	0.000171	0.003593	0	0.018644	0.136543
T4	0.003774	0.01369	0.000921	0.001014	0.000482	0.000622	0.000005	0.020508	0.143206

Similarly, the separation from the negative ideal alternative is:

$$S'_i = [\sum_j (v_j' - v_{ij})^2]^{\frac{1}{2}} \quad ; \ i = 1, \ldots, m$$

TABLE 15: SEPARATION MEASURE FROM NEGATIVE IDEAL ALTERNATIVE

Teacher	C1	C2	C3	C4	C5	C6	C7	SUM	S_i'
T1	0.003358	0.002043	0.000543	0.000146	0.000006	0.006475	0.00005	0.012621	0.112343
T2	0.01205	0	0.002562	0.001014	0.000482	0	0	0.016108	0.126917
T3	0	0.016528	0	0.000239	0.000079	0.000421	0.000415	0.017682	0.132974
T4	0.002337	0.000134	0.000411	0	0	0.003083	0.000327	0.006292	0.079322

Step -15: Calculate the relative closeness to the ideal solution C_i^* and the corresponding rank of the candidate.

$$C_i^* = \frac{S_i'}{S_i^* + S_i'} \quad ; \ 0 < C_i^* < 1$$

TABLE 16: RELATIVE CLOSENESS AND RANK OF TEACHERS

Teacher	Result	Rank
T1	0.513495	1
T2	0.453362	3
T3	0.493379	2
T4	0.356459	4

The overall ranking for each teacher is presented in Fig. 9.

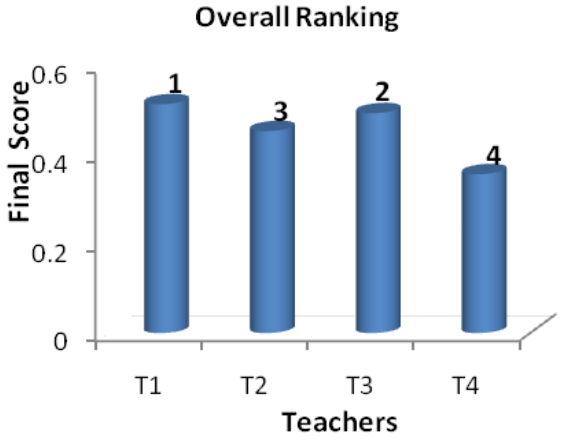

Fig.9: Overall Ranking of Teachers

V. CONCLUSION:

This paper concludes that the teacher T1 is best in his performance and followed by teacher T3 and teacher T2. The overall performance of the teacher T4 is not good enough with respect to different criteria among all other teachers. It is notable that the subject knowledge of the teacher T2 is better than the teachers T1 & T3 and T4 is also better than T3. That means it can be also concluded that in spite of having sufficient knowledge of a teacher about his subject he/she may not be the best faculty member in his department. Finally this article introduces an approach that integrates AHP with TOPSIS algorithm to support teacher evaluation decisions.

REFERENCES

[1] T. L. Saaty, The Analytic Hierarchy Process, McGraw-Hill, New York, 1980.

[2] T. L. Saaty,"Priority Setting in Complex Problems," IEEE Transactions on Engineering Management, Vol.30, No.3, pp.140-155, 1983.

[3] Christos Douligeris & Ian J. Pereira, "A Telecommunications Quality Study Using the Analytic Hierarchy Process," IEEE Journal on Selected Areas in Communications, Vol. 12, No. 2, 1994.

[4] F. Zahedi, "The analytic Hierarchy process-A survey of the method and its applications," Interfaces, Vol. 16, pp.343-350, 1977.

[5] V.S. Lai, R.P.Trueblood & B.K.Wong "Software selection: a case study of the application of the analytical hierarchical process to the selection of a multimedia authoring system," Information & Management, Vol. 25, No. 2, 1992.

[6] Maggie, C.Y.Tam, & V.M. Rao Tummala, "An application of the AHP in vendor selection of a telecommunications system," Omega, Vol. 29, pp.171-182, 2001.

[7] V.S.Akkineni & S.Nanjundasastry "The Analytic Hierarchy Process for Choice of Technologies," Technological Forecasting and Social Change, Vol. 38, pp.151-158, 1990.

[8] Debmallya Chatterjee and Dr.Bani Mukherjee "Study of Fuzzy-Ahp Model To Search The Criterion In The Evaluation Of The Best Technical Institutions: A Case Study", International Journal of Engineering Science and Technology, Vol. 2(7), 2499-2510, 2010.

[9] Pin-Chang Chen, "A Fuzzy Multiple Criteria Decision Making Model in Employee Recruitment", IJCSNS International Journal of Computer Science and Network Security, Vol. .9 No.7, pp:113-117, 2009.

[10] P. Kousalya, V. Ravindranath, and K. Vizayakumar, "Student Absenteeism in Engineering Colleges: Evaluation of Alternatives Using AHP", Journal of Applied Mathematics and Decision Sciences, Volume 2006, Article ID 58232, Pages 1–26.

[11] S. Mahmoodzadeh, J. Shahrabi, M. Pariazar, & M. S. Zaeri "Project Selection by Using Fuzzy AHP and TOPSIS Technique", World Academy of Science, Engineering and Technology 30,pp: 333-338, 2007.

[12] M.A. Abo-Sinna, & A.H. Amer, "Extensions of TOPSIS for multi-objective large-scale nonlinear programming problems", Applied Mathematics and Computation 162, 243–256, 2005.

[13] C.L. Hwang, K. Yoon, "Multiple Attribute Decision Making Methods and Applications", Springer, Berlin Heidelberg, 1981.

[14] G.R. Jahanshahloo, F. Hosseinzadeh Lotfi, M. Izadikhah, "An algorithmic method to extend TOPSIS for decision-making problems with interval data", Applied Mathematics and Computation, 2005 .

[15] Y.J. Lai, T.Y. Liu, C.L. Hwang, "TOPSIS for MODM", European Journal of Operational Research 76 (3), 486–500, 1994.

[16] Wang Y.M and Elhag T.M.S, "Fuzzy TOPSIS method based on alpha level sets with an application to bridge risk assessment", Expert Systems with Applications, 31, 309-319, 2006.

[17] Wang, Y-J and Lee, H-S, "Generilizing TOPSIS for fuzzy multiple-criteria group decision-making", Computers & Mathematics with Applications, 53(11), 1762-1772, 2007.